Society of Actuaries in Ireland

Hedging Effectiveness

15.04.14

Structure of Presentation

- Description of stochastic model
- Taylor Expansion and Delta hedge
- Underlying Volatility risk
- Realised Volatility risk
- What to hedge?
- Historic Hedging effectiveness
- P&L attribution
- CBI stress tests/Solvency II
- Practical Considerations

Stochastic Model

- 5 year put option on a stock at today's price
- 1,000 simulations with monthly intervals
- Geometric Brownian motion
- Volatility 20%
- Discount rate 2%
- Real World equity risk premium 3%
- Greeks etc. from Black Scholes formula

Some metrics of the Put Option

Metric at start of term	20% Volatility	40% Volatility
LOV (Risk Neutral)	12.5	28.3
LOV (“Real World”)	7.0	20.7
Delta	-.33	-.28
Gamma	.008	.004
CTE90 extra capital	30.6	45.8

Taylor Expansion and Delta Hedge

$$\delta Lov = \Delta\delta s + \Theta\delta t + \frac{\Gamma}{2}(\delta s)^2 + o(\delta t)$$

$$\Delta = \frac{\partial Lov}{\partial s}; \quad \Theta = \frac{\partial Lov}{\partial t}; \quad \Gamma = \frac{\partial^2 Lov}{\partial s^2}$$

$(\delta s)^2$ is $O(\delta t)$

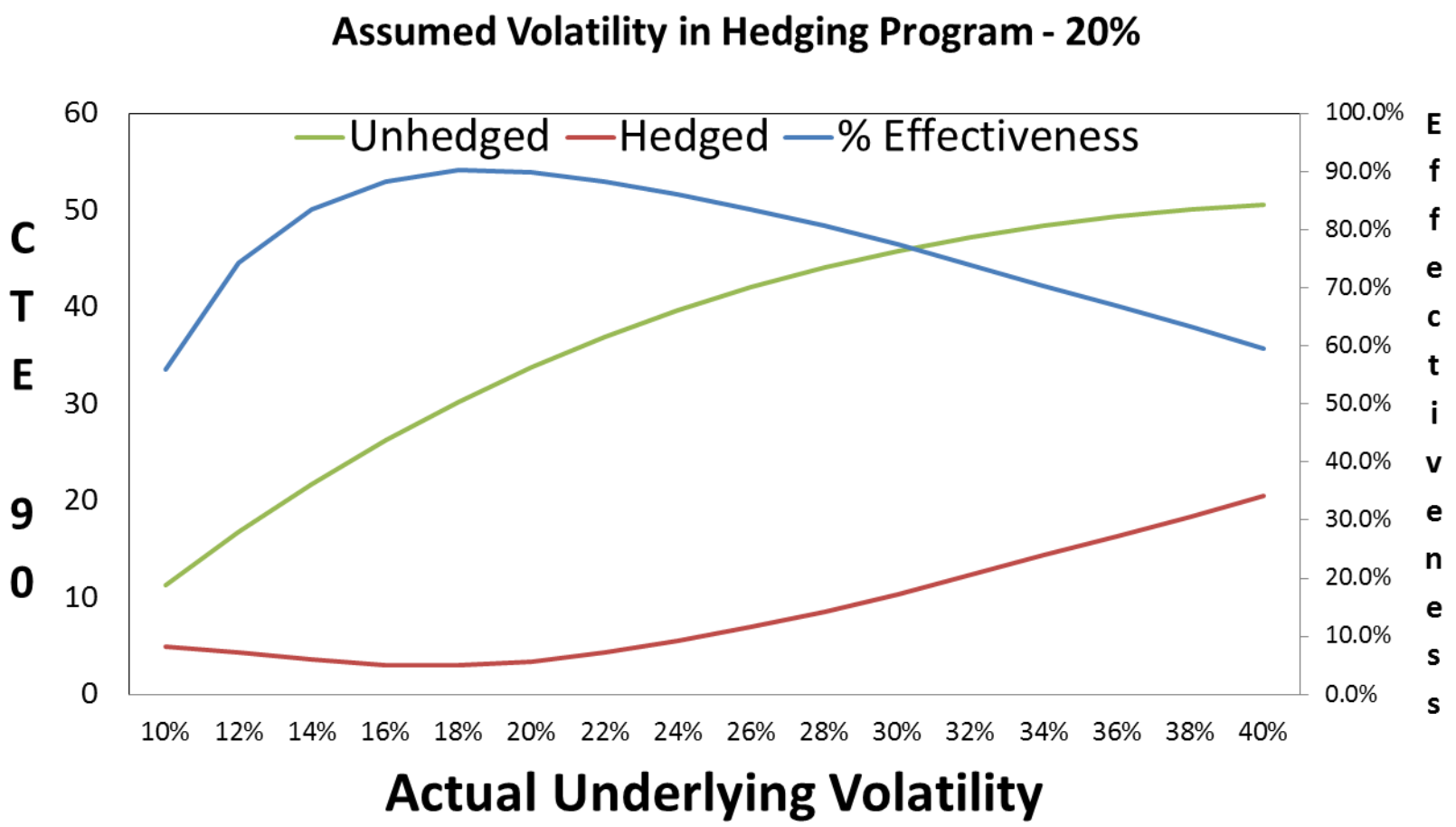
Two main risks in Delta Hedge

- Underlying volatility can only be estimated
- Rebalancing cannot be continuous

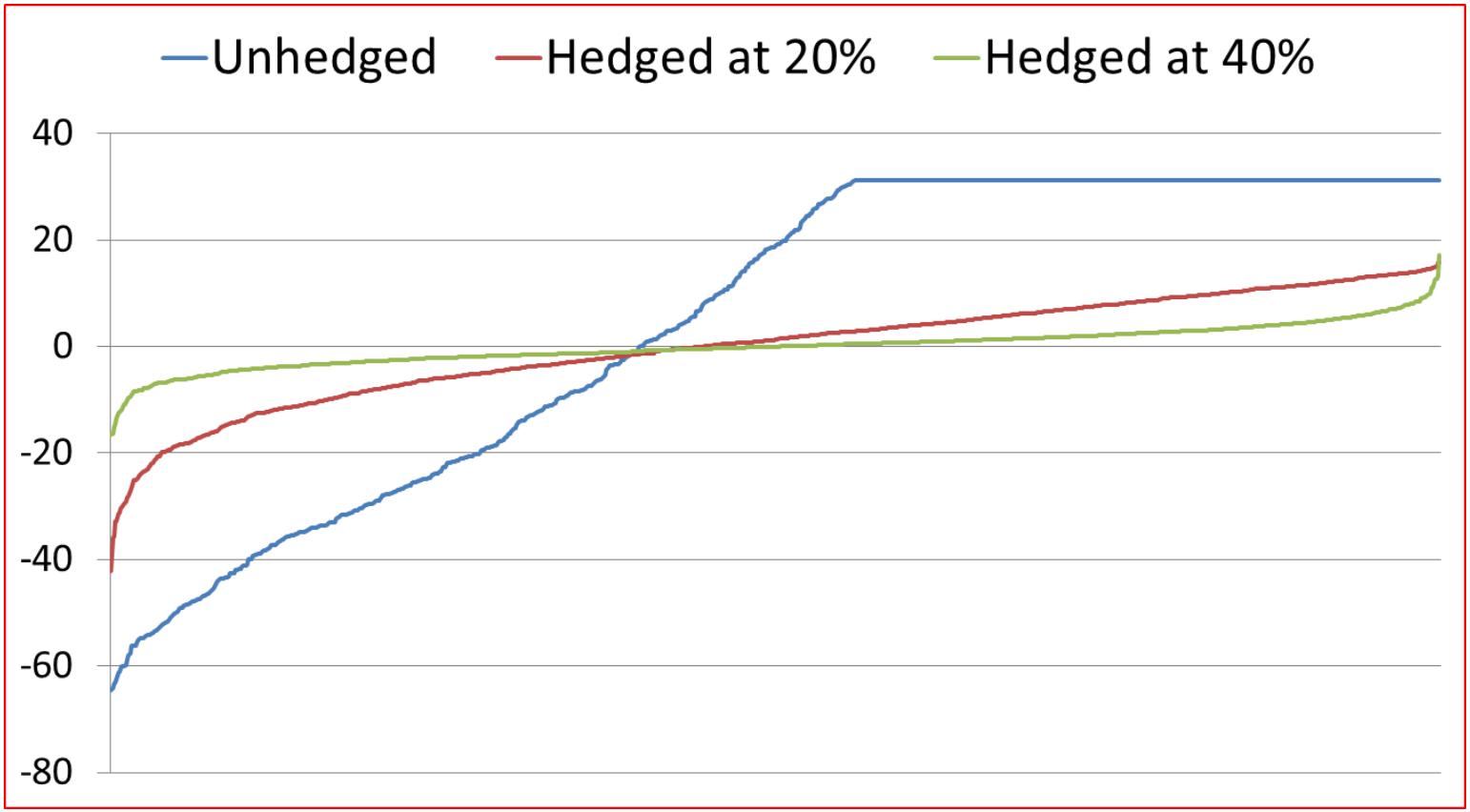
Underlying Volatility Risk

- Incorrect calculation of Delta
- Broadly symmetric but increased variability

Underlying Volatility Risk



Underlying Volatility Risk (@ 40%)



Allowance for hedging in capital

	Actual position	"Correct" position
IFRS reserves	*12.5*	*28.3*
Extra gross capital	*30.6*	*45.8*
Allowance for hedging	*-27.5*	*-27.2*
Combined Reserves + Capital	*15.1*	*46.9*

Underlying volatility 40%
Volatility assumption in hedging, IFRS and capital calculation 20%

Targeting Volatility

Synthetic index of risky asset and cash

Exposure to risky asset = Target Volatility/Realised Volatility

Possibly subject to a Cap

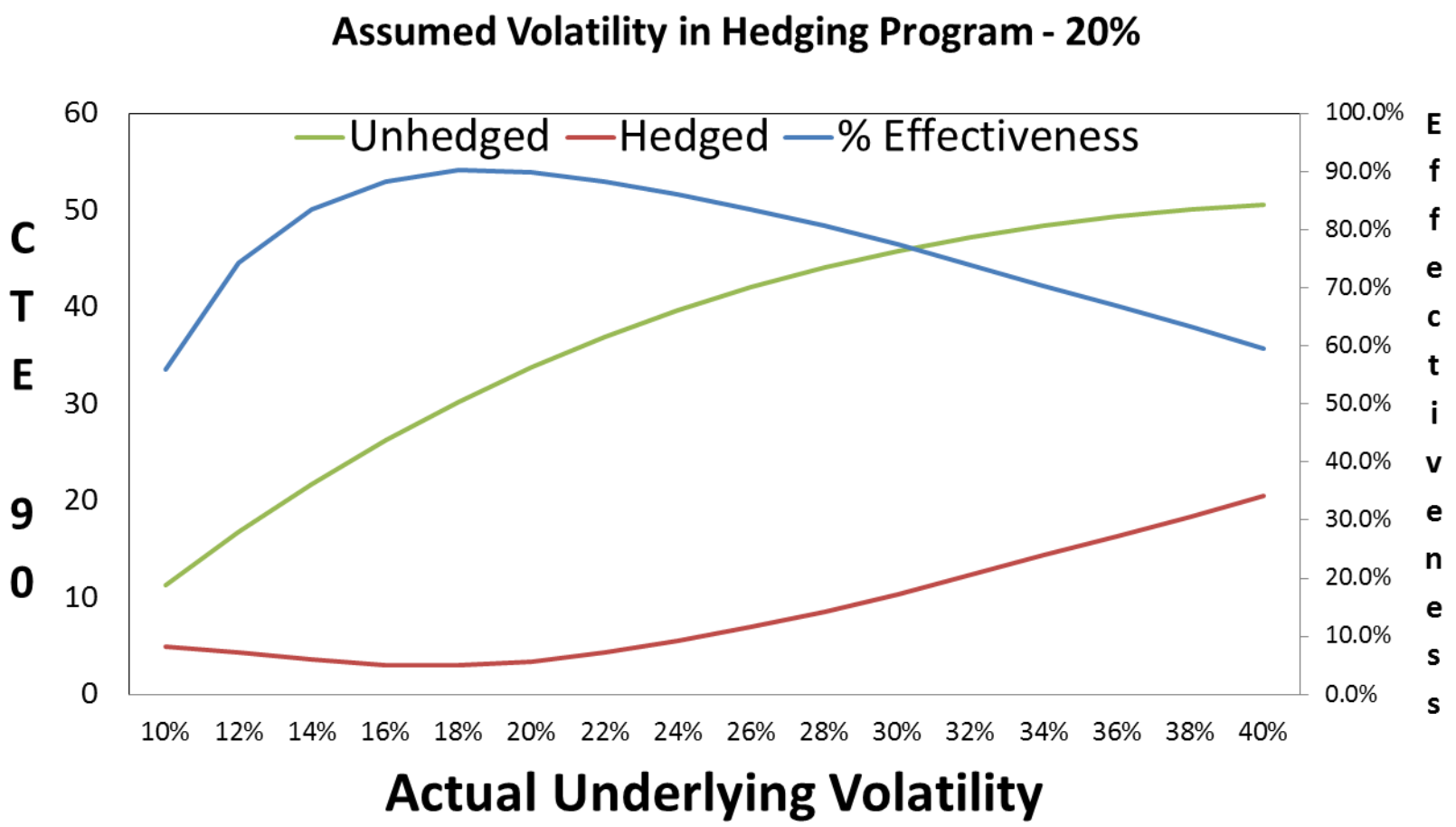
Realised Volatility measured over last 20 days, say

Realised Volatility estimator of Underlying Volatility

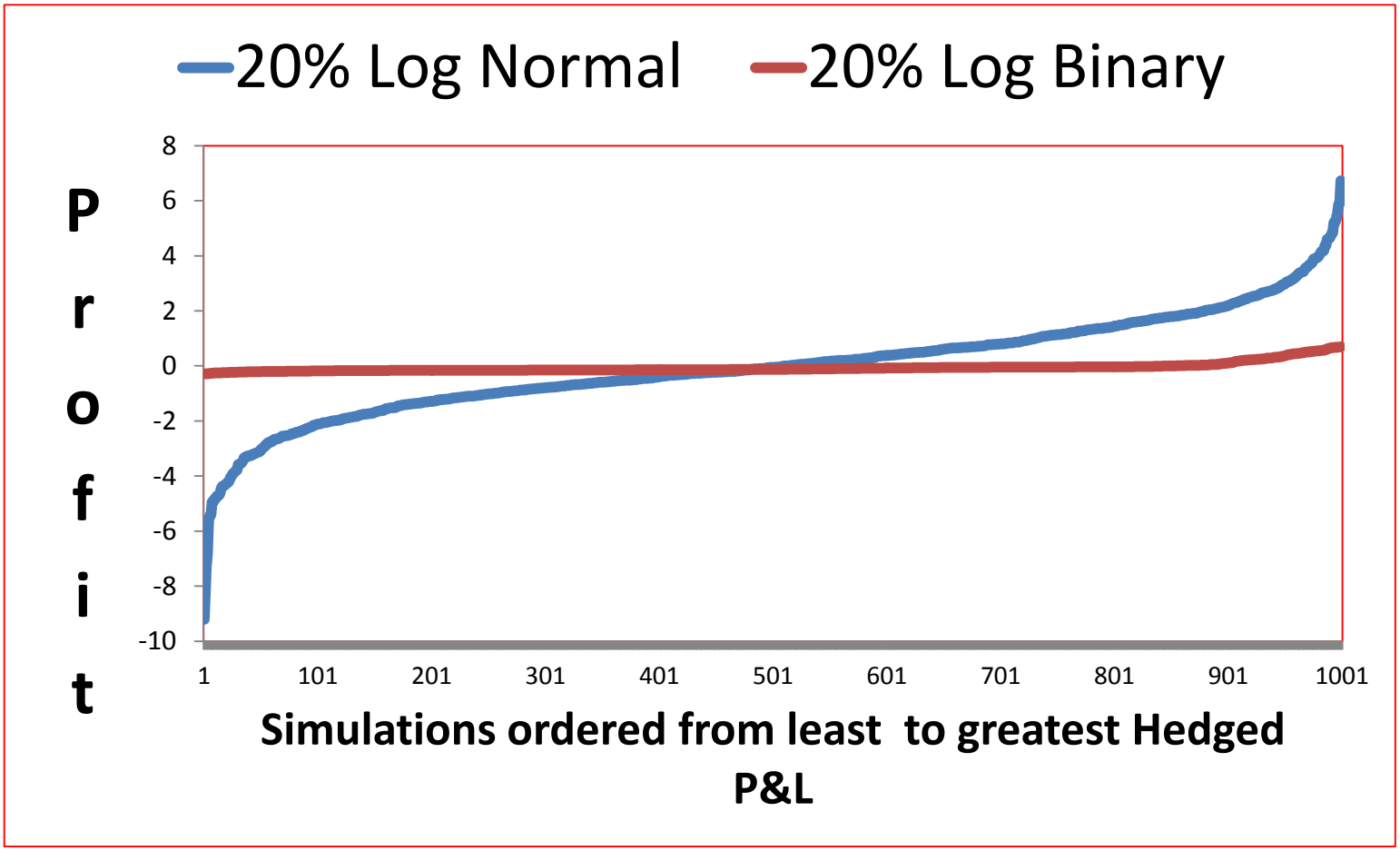
Targeting 20% Volatility

Cap	Underlying Volatility		
	15.0%	20.0%	30.0%
100.0%	15.0%	19.1%	20.9%
110.0%	16.4%	20.0%	21.0%
120.0%	17.7%	20.5%	21.0%
130.0%	18.8%	20.7%	21.0%
140.0%	19.6%	20.9%	21.0%
150.0%	20.1%	20.9%	21.0%
No Cap	21.0%	21.0%	21.0%

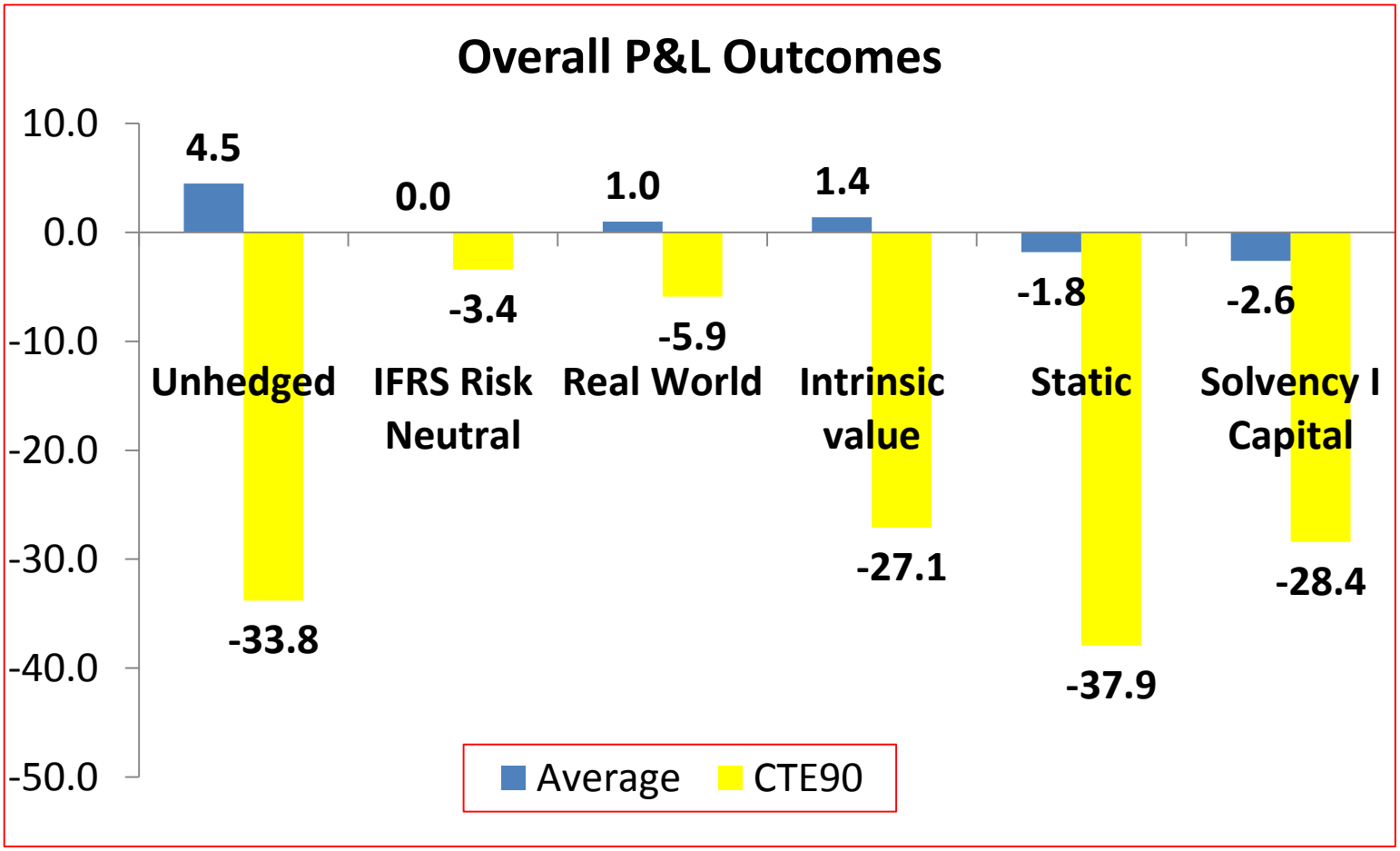
Underlying Volatility Risk



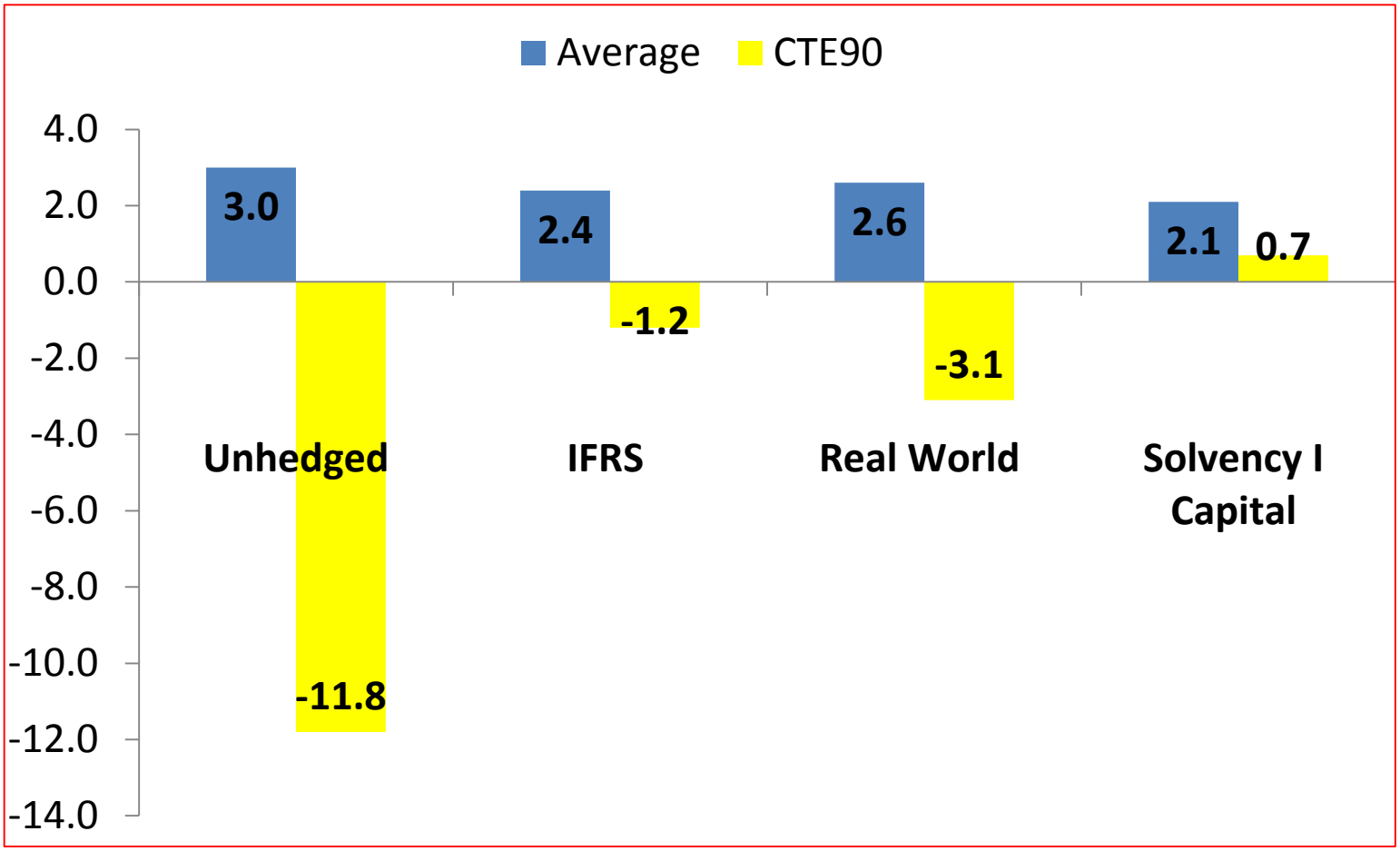
Realised Volatility Risk



What should we Hedge?



One Year View (Solvency I Capital)



Historic Hedge Effectiveness

"The extent to which the hedging programme is projected to offset total guarantee liability movements ***is not to exceed that based on a credible amount (2 years or more) of historical data****."*

Stress Testing framework for the Variable Annuity Industry *CBI 2013*

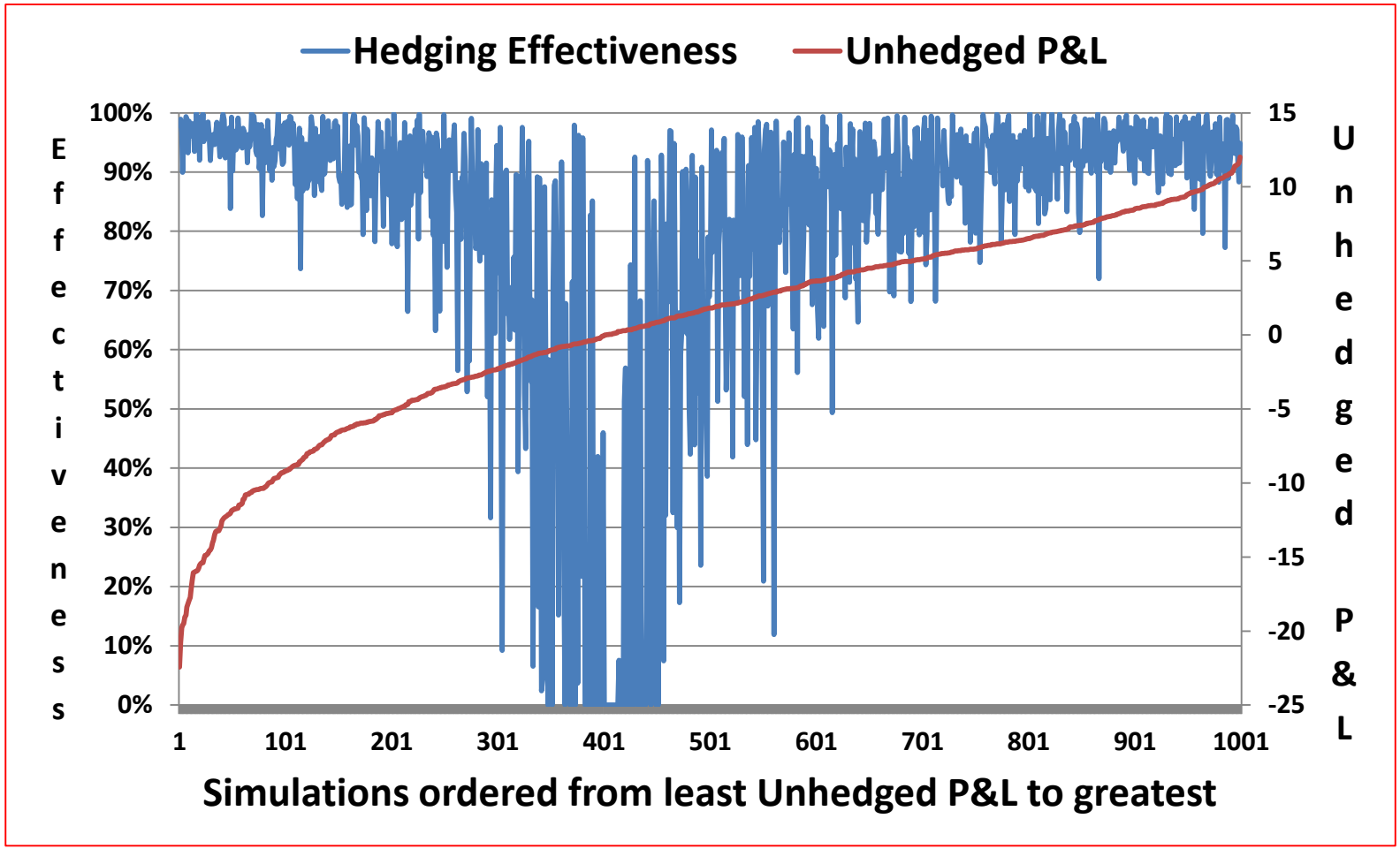
Historic Hedge Effectiveness

- Define historic hedging effectiveness as follows:

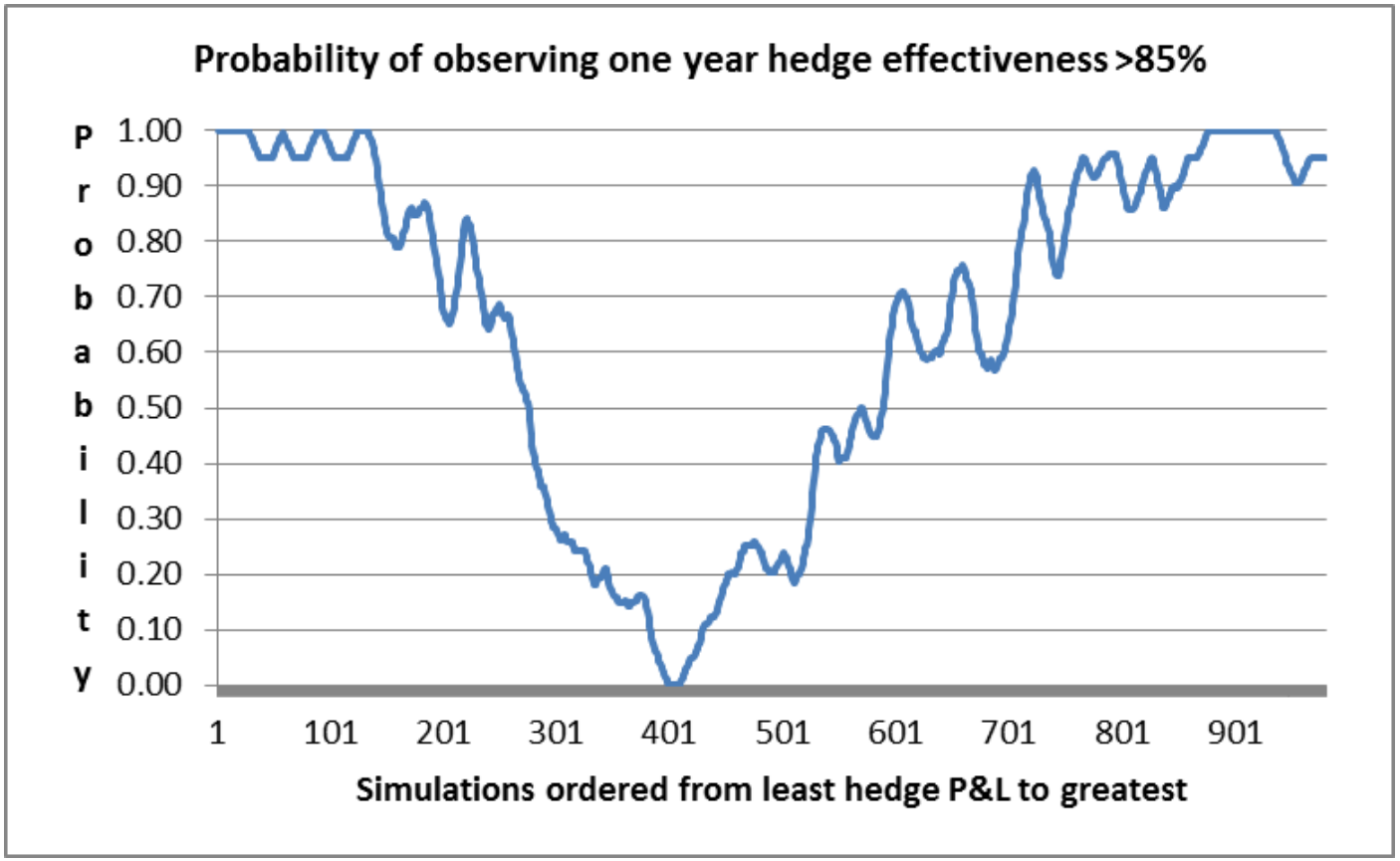
$$\left[1 - \left|\frac{Hedged\ P\&L}{Unhedged\ P\&L}\right|\right]^{+}$$

- Conceptually different from prospective version

Historic Hedge Effectiveness One Year



Historic Hedge Effectiveness



Hedging Rho (interest rate)

- Duration matching
- Optionality hedging
- Duration matching highly effective
  - Possibly flatters historic hedge effectiveness

P&L Attribution

$$\delta Lov = \Delta\delta s + \Theta\delta t + \frac{\Gamma}{2}(\delta s)^2 + o(\delta t)$$

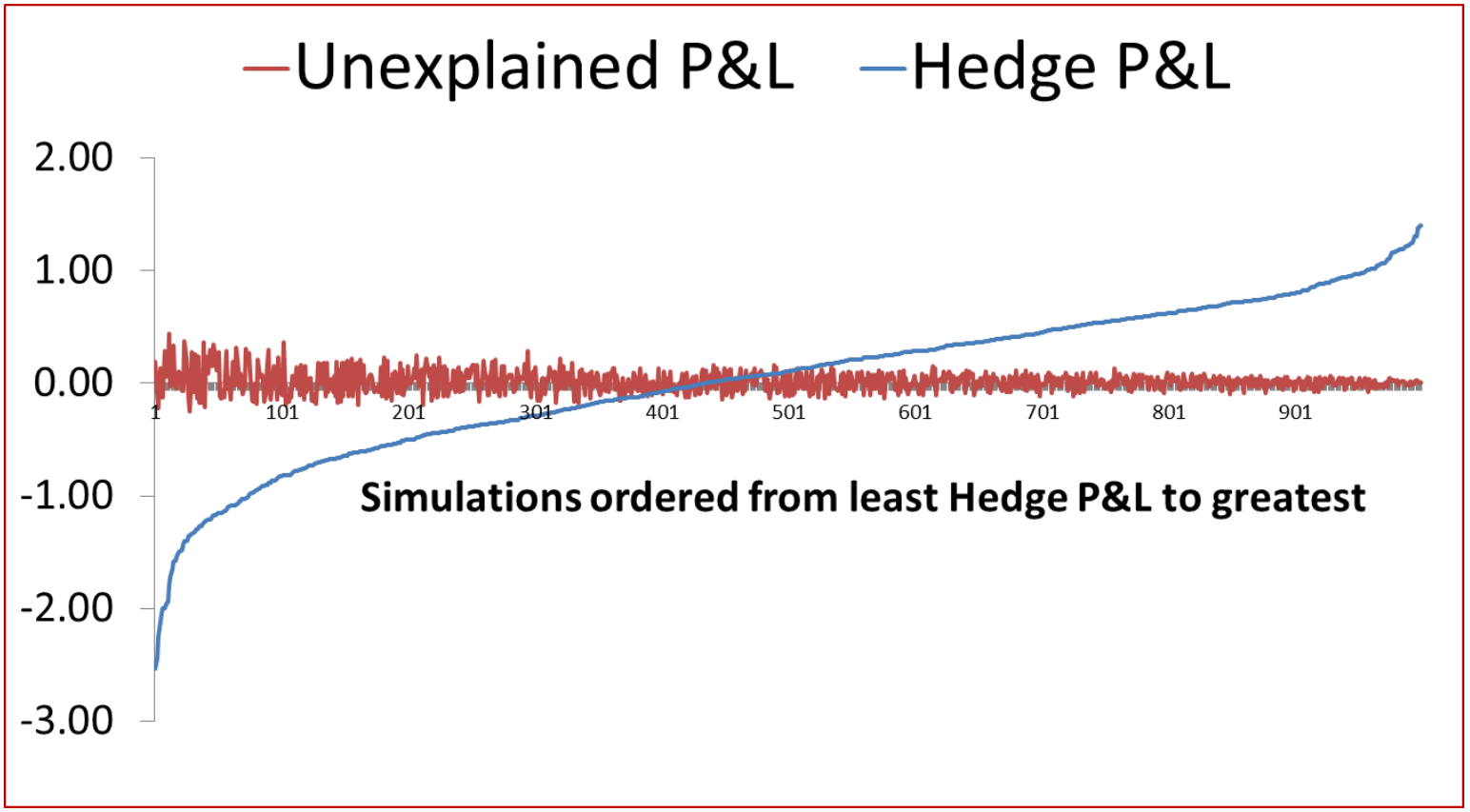
$$< \delta Lov >= \Delta < \delta s > + \Theta\delta t + \frac{\Gamma}{2} < (\delta s)^2 > + o(\delta t)$$

$$\delta Lov - < \delta Lov >$$
$$= \Delta(\delta s - < \delta s >) + \frac{\Gamma}{2}((\delta s)^2 - < (\delta s)^2 >) + o(\delta t)$$

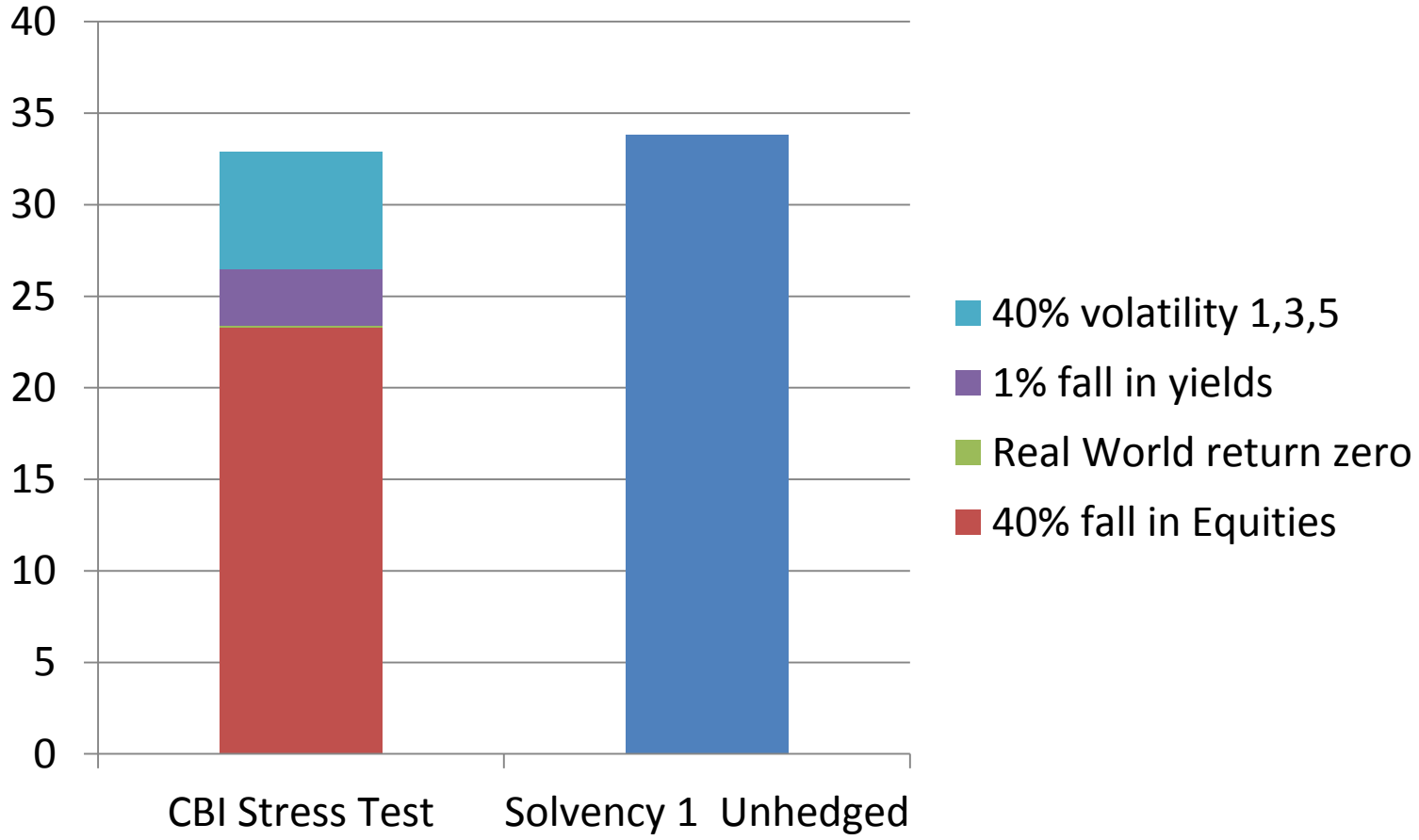
If fully delta hedged:

$$\boldsymbol{P\&L = \frac{\Gamma}{2}((\delta s)^2 - < (\delta s)^2 >) + o(\delta t)}$$

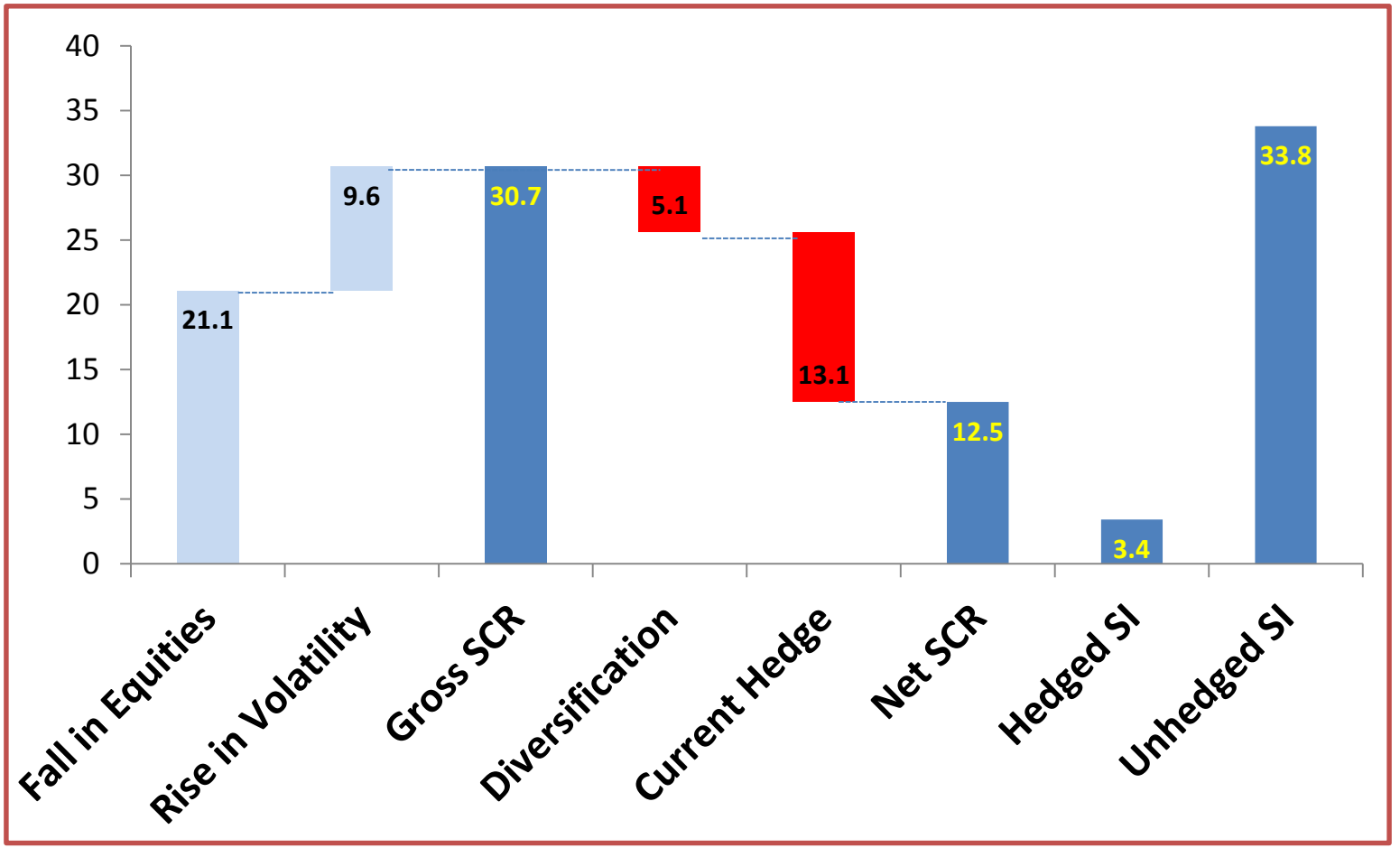
Unexplained/Residual P&L



CBI Stress Tests



Solvency II


40
35
30
25
20
15
10
5
0
21.1
9.6
30.7
5.1
13.1
12.5
3.4
33.8
Fall in Equities
Rise in Volatility
Gross SCR
Diversification
Current Hedge
Net SCR
Hedged SI
Unhedged SI

Practical Considerations

- Demographic and Behavioural considerations
- Long term and possibly regular premium
- Hedging interest rates important for GMIBs
- Basis risk
- Volatility capping and targeting
- P&L attribution should allow for “cross greeks”
- Product charges
- Transaction costs.

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