

# **RBC and its Practical Applications to Non-Life Insurance**

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### 1.0 Introduction

The title of this paper is “RBC and its Practical Applications to Non-Life Insurance”. RBC, or Risk Based Capital, is a very wide and diverse area. No single paper could do justice to the complexity and extent of the subject. The main purpose of this paper is to provide an outline of how RBC models can be used to provide valuable insight into your business and give senior management practical tools to help make informed decisions.

§2 of the paper introduces the concept of “Capital”. This section also outlines the risks that should be modelled and discusses briefly the assumptions necessary.

§3 describes a possible approach to calculating the Risk Based Capital (RBC) figure for a non-life insurance company.

§4 considers how you can apply your RBC model to produce an appropriate allocation of capital to business lines, while §5 looks at how you might use an RBC model to help with reinsurance decisions.

In §6 other practical problems such as reserving, and optimal decision making are touched upon.

Finally, §7 describes a number of practical issues that you might face when trying to implement a model and apply it to business problems.

Throughout the paper reference will be made to an example company that will be used to illustrate the issues being discussed. This company writes three lines of business:

- Motor
- Home
- Liability

Full details of the company and the assumptions underlying the results are provided in Appendix A. Note that the characteristics of the example company’s lines of business have been chosen to allow us to illustrate some relevant ideas in this paper, and do not necessarily reflect current conditions in the Irish non-life insurance market.

## 2.0 Capital & Risks

### 2.1 Capital

What do we mean by “capital” and why does an insurance company need capital? Various terms are used in conjunction with “capital”; these terms and their meanings as used in this paper are defined below.

**Table 2.1.1: Definitions of Capital**

Term	Definition
Capital	The margin of assets over liabilities.
Regulatory / Solvency / Statutory Capital	The amount of capital that an insurance company must hold to satisfy the solvency requirements of its regulator.
Available Capital	The actual margin of assets over liabilities that an insurance company holds on its balance sheet.
Risk Based Capital / RBC	Capital is “risk based” if it is determined in a scientific way, taking into account the actual risks that a company faces.

Capital is needed to absorb unexpected changes in the values of assets and liabilities so that a company can remain solvent. In order to determine the RBC we need to use an Asset Liability Model (ALM) of the company that projects future cashflows, profitability and solvency allowing for the interaction of risks that the company faces. This immediately raises a couple of key questions:

- What risks should be modelled?
- How should the model be structured?

§2.2 describes the risks to be modelled. The ALM itself consists of a series of interacting modules or models, such as the asset model, the reserving module, the catastrophe model etc. For a more detailed description of a possible model structure see Appendix B.

### 2.2 Risks to be Modelled

The new Solvency II regime and the current FSA regime in the UK provide a template of the risks that should be considered by an insurance company. These are as follows:

- Underwriting risk
  - Premium risk
  - Reserve risk
  - Claims risk (individual large claims and accumulation of attritional claims)
  - Catastrophe (CAT) risk

- Market risk
  - Equities
  - Bonds
  - Property
  - Currency exchange rates
- Credit risk
  - Reinsurance
  - Default on bonds
  - Other counter-party risk
- Operational risk

### 2.3 Assumptions

It is not in the scope of this paper to explore in detail the setting of assumptions. However it is useful to make some observations.

#### 2.3.1 Key assumptions

Economic assumptions will normally be set by the use of an Economic Scenario Generator (ESG), which provides an internally consistent set of economic scenarios to drive the asset model. The outputs of the ESG will also interact with the assumptions for other aspects of the ALM, for example claims inflation and new business volumes.

- Claims assumptions
  - Catastrophe claims from the CAT model.
  - Large claims frequency and severity.
  - Attritional claims (i.e. all claims other than Cat claims and Large claims) frequency and severity.
  - Claims inflation assumptions.
- Expenses
  - Per policy expenses.
  - Claim expenses.
  - Commission.
  - DAC %.
  - Expense Inflation assumptions.
  - Expenses in run off.
- Patterns
  - Premium / exposure earning pattern.
  - Claim payment patterns.
- Underwriting cycle
  - Volatility of premium rates.
  - New Business Volumes.

- Correlations
  - Between classes.
  - Between assumptions.

### 2.3.2 Realism

A key theme when setting assumptions is realism. If you do not use realistic assumptions then your results will be biased and less meaningful. There is no rationale for being prudent or even erring on the side of caution.

### 2.3.3 Spread

In traditional actuarial techniques a lot of focus is given to the mean value. For risk based capital calculations, and indeed any investigations that require stochastic simulation, it is vital that particular attention is also given to the spread (or standard deviation) of the distribution.

### 2.3.4 Parameter Uncertainty

We need to remember that assumptions are usually derived from the company data as well as a big dose of “actuarial judgement”. We can not be certain that the model, distribution and parameters that we have fitted to the data will be a true representation of the underlying behaviour of the variables that we are modelling.

This is especially true when there is little data to work with, such as large claims or catastrophe claim events and leads to the concept of parameter uncertainty.

The problem of parameter uncertainty is particularly challenging in the context of RBC, which by definition examines the extreme tails of distributions where there is very little data to base an analysis upon. In fact some commentators suggest that it is not possible to rely on the extremes of the distributions of results because of the paucity of data.

### 2.3.5 Input Parameters

A key input to the model will be the starting position. This includes

- Balance sheet (For Total Company and by Line of Business)
  - Statutory
  - Realistic
- Reserve data
  - Starting Position for Reserve triangles
- Premium
  - Premium written in past year
  - Exposure written in past year

## RBC and its Practical Applications to Non-Life Insurance

Most of the information required should be readily available from the usual financial sources. One notable exception is the “realistic balance sheet”. The statutory balance sheet is converted into the realistic balance sheet by removing all margins from the assets and liabilities. Appendix A provides a description of how to convert the statutory balance sheet into a realistic balance sheet. The balance sheet must be presented for the company in total and split by class of business. Again Appendix A sets out how you could split the balance sheet between the different classes of business.

### 3.0 Calculation of Risk Based Capital

This section describes how to calculate an RBC figure. We will use our example company as a practical illustration.

#### 3.1 What is RBC?

To determine the RBC figure we need to project the future solvency position of the company and determine how much capital is required to ensure solvency at a given probability. This can be compared to the existing available capital to ascertain the current level of cover. Risk Based Capital is concerned with the realistic solvency position of the company - as distinct from the statutory solvency position. This realistic solvency position will reflect the removal of any unnecessary margins relative to the statutory position. Appendix A explains how the statutory balance sheet is converted to a realistic one.

For each simulation the model will project the profit & loss account and corresponding balance sheet at each year-end on both a statutory and realistic basis. The projections are also performed for individual business lines as well as for the company as a whole. Analysis of the simulations will be used to determine how much capital the company must hold to satisfy its solvency requirements. For the purposes of this paper RBC can be thought of as the excess capital required now to ensure, at a given confidence level, that the company will have sufficient assets to meet all liabilities at every point over the entire projection period. This means that if a given simulation results in insolvency at time 't', but subsequent future profits lead to a future solvent position, the capital requirement is taken as that at time 't'.

#### 3.2 A Technical Definition of RBC

The technical definition of RBC will need to incorporate the following items:

- Time horizon  
The period in the model projection for which the company is open to new business and renewals.
- Run off  
Assumed the company to be closed to new business, how long will the model allow for the reserves to be run off.
- Risk tolerance  
In simple terms this corresponds to probability of ruin we are willing to accept.
- Risk measure  
What method do we use to convert our concept of "risk" to a number?
- Capital flows  
Assumptions regarding payment of dividends or raising debt or equity capital.

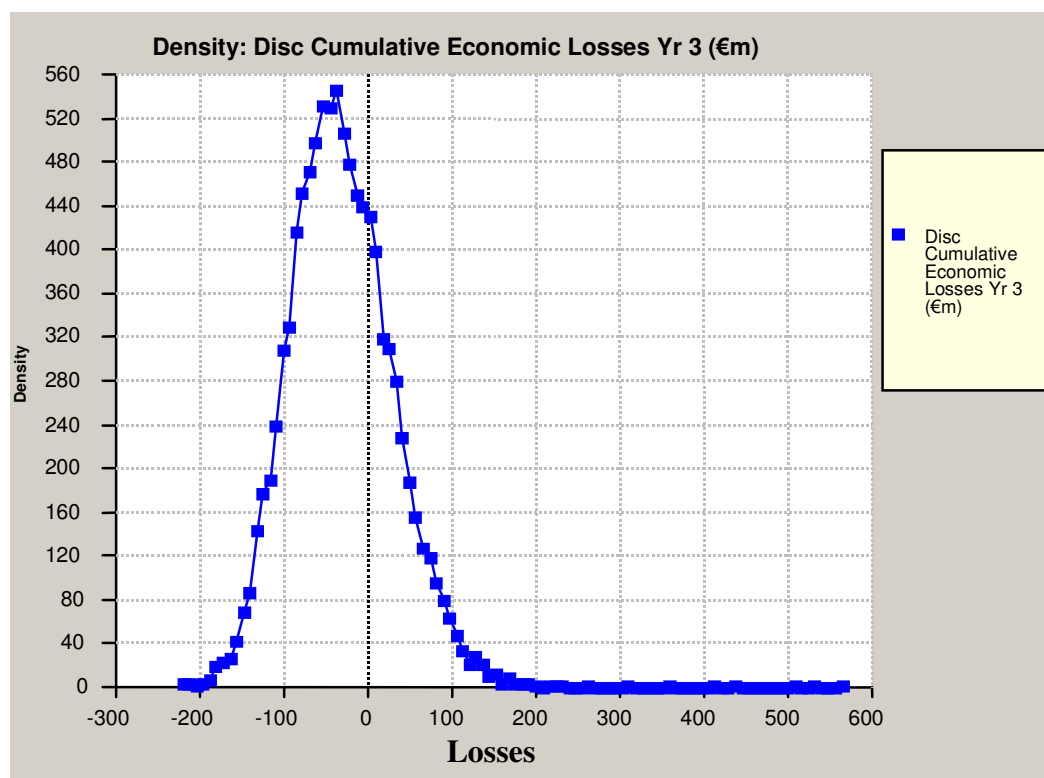
Once the model is constructed and all the parameters are in place we are ready to run the model. A very large number of simulations is required to

minimise the effect of sample error. The RBC will be calculated from the simulated “Minimum Discounted Net Economic Assets” at each projection period. The term is explained as follows:

- “..net..assets” - this is the surplus of assets over liabilities (may be negative).
- “..economic..” - another word for realistic.
- “..discounted..” - cashflows are discounted using the return on assets achieved in the particular simulation. The RBC is the amount of capital we require now to ensure solvency in the future. As such we need to allow for the potential future gains / losses in these assets in each simulation over the appropriate projection period.
- “minimum...” - the minimum value is carried forwards so that future positive cashflows cannot be used to reduce current deficits. As described early this implies solvency at all times in the projection.

For illustrative purposes the probability density function implied by the 10,000 simulations for ‘Discounted Cumulative Economic Losses in Year 3’ is illustrated below in Figure 3.1.1. It will become clear a little later why year 3 was chosen.

**Figure 3.2.1: Discounted Cumulative Economic Losses at the end of Yr 3**

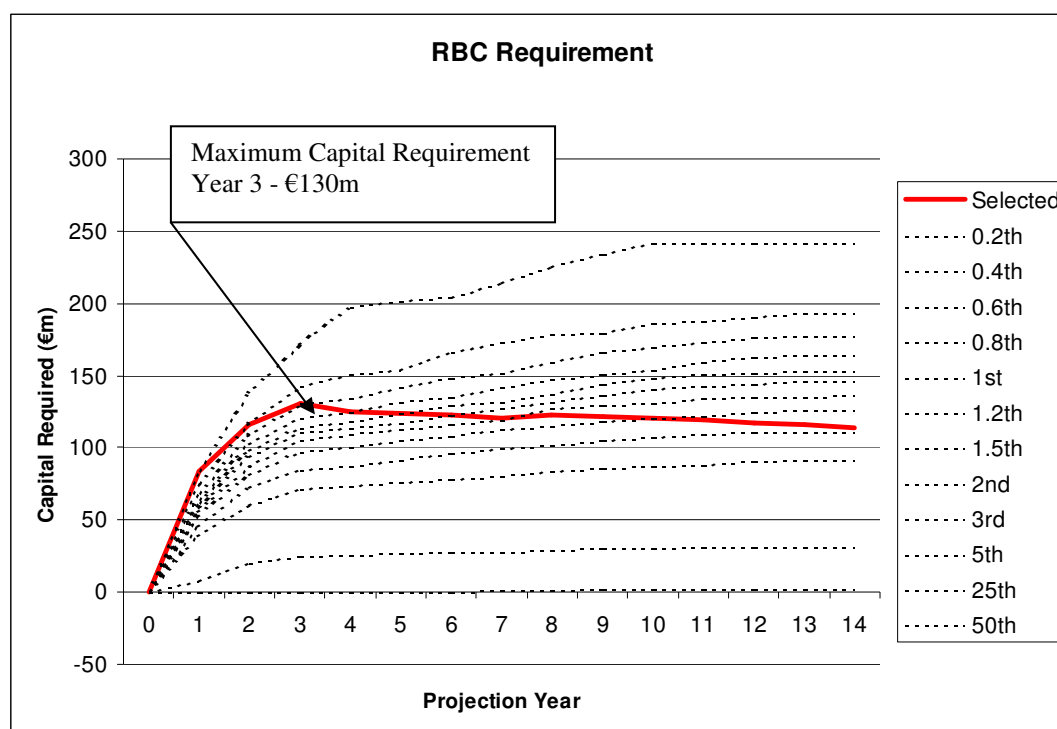


### 3.2.1 Risk Tolerance

The risk tolerance is the required probability of remaining solvent over the term of a projection. In practice for a risk tolerance of say 99.5% with 10,000 simulations, we would require the company to be solvent in all but the 50 “worst” simulations.

The dotted lines in Figure 3.2.1.1 below show the capital required at future year-ends at various tolerance levels. In each case the capital required is the amount of capital needed at time zero so that the company remains solvent at all times, up to and including time t. It is important to understand that the dotted lines do not necessarily represent any single simulation, rather they provide for each year the result of the particular simulation given the tolerance level in that year. Thus the dotted lines increase monotonically over time.

Figure 3.2.1.1: Illustration of Variable Risk Tolerance



It follows therefore that if we were to use a constant risk tolerance, we could just look at the final year of the projection and set our capital requirement based on that year alone.

However in practice if we were to use too tight a risk tolerance, over too long a time period, we would have an impractically onerous capital requirement given the greater and greater uncertainty of the results as we move further in time (the “expanding funnel of doubt”).

The practical response to this problem is to use a risk tolerance that reduces over time. In our model we have used a risk tolerance of 99.8% in year 1

reducing by 0.2% for each subsequent year. The red line in figure 3.2.1.1 above represents this.

The combination of increasing uncertainty and reducing tolerance level means that the capital requirement peaks after several years. In our example this happens at the end of year 3 as illustrated in the graph above. This year with the peak capital requirement is known as the “driver year”. This is the relevance of Year 3 referred to in figure 3.2.1 above.

### 3.2.2 Risk Measure

Risk Measures are discussed in more detail in Appendix C. Two common measures are VaR and tVaR. These measures look at the tail of the distribution of outcomes. VaR looks at a particular percentile of the distribution whereas tVaR looks at the conditional expected value of the distribution, given that the value exceeds a particular percentile. We have attempted to illustrate these graphically below.

Figure 3.2.2.1: VaR at driver year

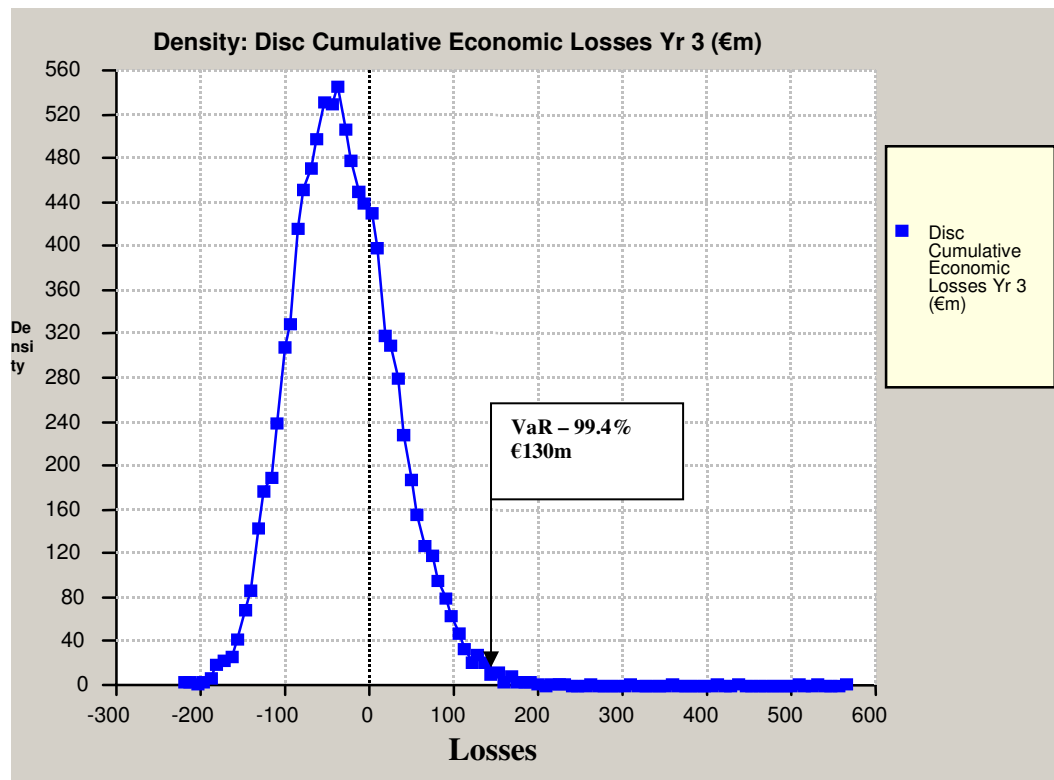
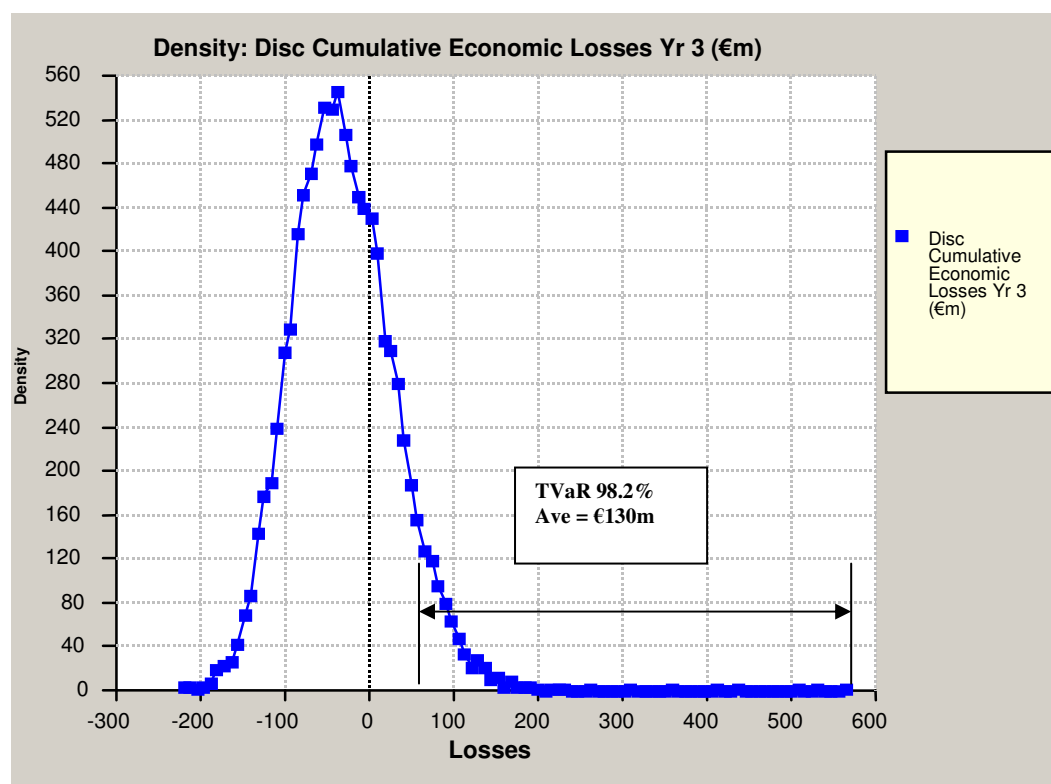


Figure 3.2.2.2: tVaR at driver year



### 3.2.3 Capital Flows

At different times companies may increase or reduce the amount of available capital. Payment of dividends will reduce available capital, whereas raising debt or equity could increase available capital.

The model could be constructed so that dividends are paid when the company can afford to and capital is raised when it is needed. Alternatively the model could ignore all such capital flows.

The definition of RBC is the amount of capital required now to prevent future calls on capital resources. Furthermore, the RBC deals with extreme loss-making scenarios where it is unlikely that a company could pay dividends. These arguments would suggest that capital flows are ignored.

On the other hand a company may have a strict dividend policy where it would be more realistic to model the capital flows.

For our example company we ignore all capital flows.

### 3.3 Risks Included

Before revealing the RBC result for our company it is important to document which risks are included.

- Underwriting risk
  - Premium risk
  - Reserve risk
  - Claims risk (individual large claims and accumulation of attritional claims)
  - Catastrophe (CAT) risk
- Market risk
  - Equities
  - Bonds
  - Property
  - Currency exchange rates
- Credit risk
  - Reinsurance
  - Default on bonds

Our model *does not* include

- Operational risk

### 3.4 RBC Result

The final RBC result will depend on the combination of risk measure, risk tolerance, time horizon and run-off period chosen. For our worked example the calculation is based upon

- Time horizon            3 year (do not confuse this with driver year)
- Run off                    10 years
- Risk tolerance           99.8% in Year 1 reducing by 0.2% per year.
- Risk measure            VaR
- Capital Flows            Assume there are no capital flows

So in words the RBC is:

The minimum amount of capital that should be held today to be at least 99.8% sure of economic solvency up to the end of year 1, 99.6% sure of solvency up to the end of year 2, and so on with the probability reducing by 0.2% per annum over the next fourteen<sup>1</sup> years.

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<sup>1</sup> 3 Years New Business + 1 Year to Earn + 10 Years to Run-Off gives 14 Years in total.

Summary of RBC Result

Table 3.4.1: RBC for Company as a Whole (€m):

Company	RBC
Statutory Capital	99.7
Economic Capital	188.3
RBC	130.1
RBC as % of yr 1 GWP	45.4%
Economic Assets in excess of RBC	58.2

The graph below helps to illustrate the contribution of each class to this RBC requirement.

Figure 3.4.1: Results of 30 Simulations around RBC by Class of Business

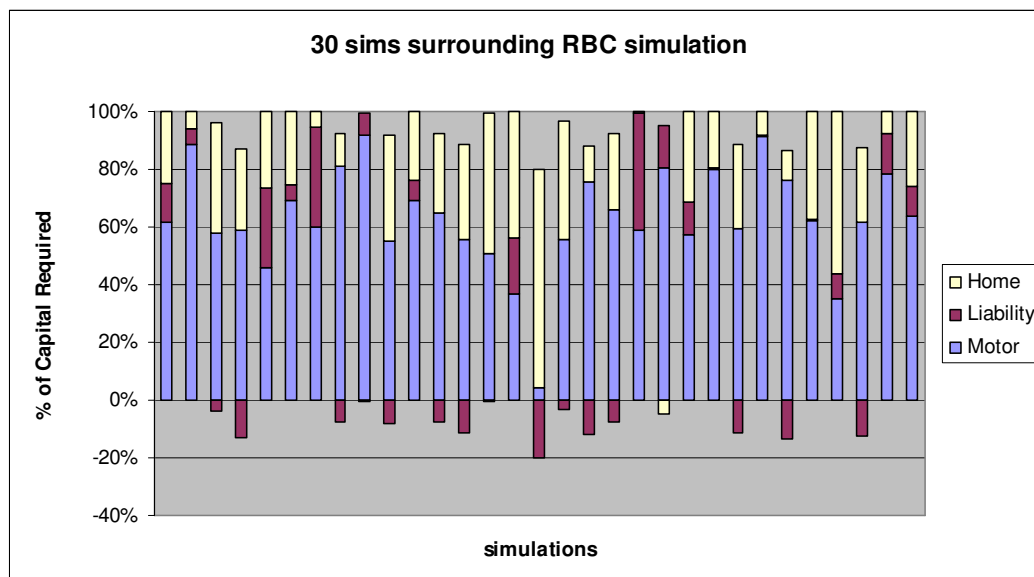


Figure 3.4.1 above uses the results at the end of year 3. Each bar represents an individual simulation. For each bar the proportion of the loss caused by each class of business in that simulation is illustrated. A negative percentage of capital required for a class within a simulation signifies that in this instance this class is actually making a profit.

Some interesting points to pick out from the above diagram are:

- The class requiring the most capital is Motor followed by Home and Liability respectively.
- In a few instances Liability is actually contributing capital.
- The very large capital requirement for Home is driven by CATs (if we looked at the worst 10 simulations then the bars are heavily dominated by Home driven by CATs - bear in mind for the figure above we are looking at the simulations around the RBC result, with the RBC result being the 60<sup>th</sup> worst simulation).

If we notionally ‘ring-fence’ each class of business we can calculate standalone RBC’s for each class. In this instance for any individual simulation losses in one class are not offset by profits in another. These resulting figures are illustrated in the table below.

**Table 3.4.2: Standalone RBC By individual line of business (€m)**

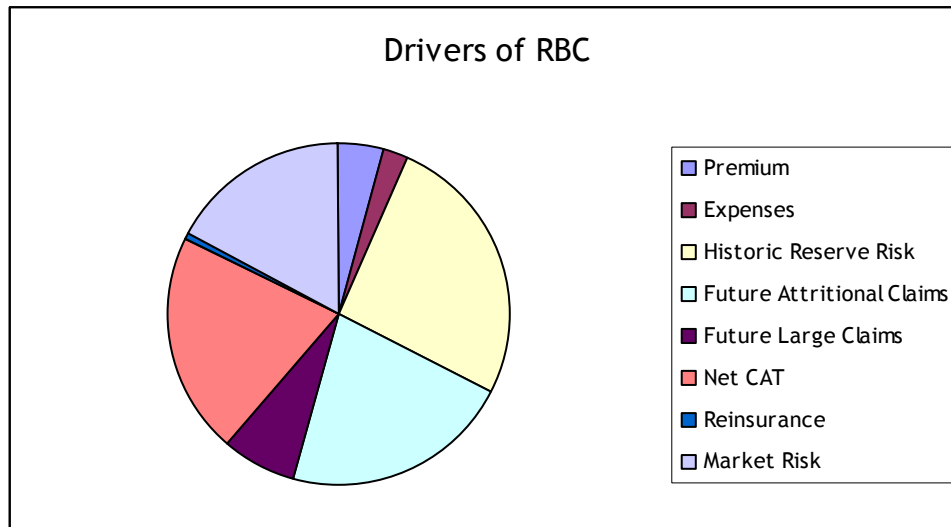
Class	Standalone RBC
Motor	104.5
Home	63.8
Liability	35.7
<b>Total</b>	<b>204.0</b>

Note the difference between the RBC for the company as a whole and for the individual lines of business. This provides an indication of the level of diversification benefit. In our example the diversification benefit is €73.9m.

### 3.5 What Drives the RBC Requirement?

As well as analysing the result by class of business we can look at the actual cash flows that drive the RBC result. Figure 3.5.1 below illustrates the component parts of the projected cash flows that produce the RBC figure

**Figure 3.5.1: Drivers of RBC**



The cash flows have been shifted from their expected positions to provide a measure of their actual volatility. It is clear from the figure that the four key drivers are historic reserve risk, future attritional claims, catastrophes (net of reinsurance) and market risk.

## 4.0 Capital / Risk Allocation

### 4.1 Introduction

The description of the model in Appendix B stresses the importance of retaining the information and results at a suitably detailed level. With this structure it is possible to look at Risk Based Capital at the line of business level. This leads us to the concept of Capital / Risk Allocation.

It is important in designing the model to build it in such a way as to be able to tackle the issues described in this section. It is easiest to introduce the ideas involved by way of a case study using our example company.

### 4.2 A brief pause for thought

Before moving to the case study it is worth briefly discussing

- (i) Why allocate Risk / Capital in the first place?
- (ii) What is a Risk Measure and what properties should a suitable Risk Measure / Capital Allocation methodology have?
- (iii) How should we allocate diversification benefit?

#### 4.2.1 Why Allocate Risk / Capital?

A prime reason to attempt an allocation of capital to lines of business is to allow management to assess whether individual lines of business are contributing adequately to overall profitability. This question can be answered by measuring the return on capital by line of business and comparing this to the cost of capital (required return on capital) appropriate to the line of business. In order to measure the return on capital it is first necessary to allocate capital to each line of business.

A devil's advocate may argue that any results are based on subjective assumptions and dubious distribution fitting, or that the methodologies focus solely on the tail where there is very limited information. However in defence of capital allocation it is clear that it adds to our understanding of the different risks and the volatility inherent in the various lines of business.

Armed with an enhanced understanding of risks we can use Risk Allocation to ensure that pricing for a given line of business allows for risk appropriately, and at a higher level to ensure that business strategy is aligned with use of capital. In any case, the techniques involved will be a necessary part of our arsenal if only because Regulatory Capital Requirement will be based upon the same methodologies.

Note that it is not intended to provide a detailed review of capital allocation, but rather to highlight the types of issues to be considered in constructing a model that will be used for capital allocation.

### 4.2.2 What is a Risk Measure?

We have already encountered risk measures in §3.2, where we saw a risk measure used to set the RBC for the company as a whole. The same concept can be extended to apply by line of business, but there are some desirable properties we would wish our chosen risk measure to satisfy when we begin to drill down below the total company level.

These properties are described under the umbrella of “coherence”. The idea behind coherence is that we can sensibly split/recombine the company into parts and avoid any anomalies when we apply the risk measure to the parts and the whole.

A more detailed discussion of risk measures and the coherence property is given in Appendix C.

### 4.2.3 Allocating Diversification Benefit

We have seen in our example company that there is a substantial diversification benefit, that is the aggregate RBC for the whole company is less than the sum of the RBC’s for the individual lines of business.

Capital allocation methods aim to allocate this diversification benefit to the lines of business in a way that reflects the risks inherent in each line and the correlations between them. The concept of coherence again arises here and is closely related to the coherence of the risk measure used.

Appendix C discusses some of the theoretical methods for allocating diversification benefits in more detail.

### 4.3 Capital Allocation Case Study

The discussion of capital allocation below takes the form of a case study. In this study we start with a relatively simple methodology and then examine each assumption individually to arrive at a (hopefully) more satisfactory final methodology. The case study uses the same example company described in Appendix A, so our starting point is the question; “how do we allocate the €130.1m to the three classes of business?”

For the purposes of this paper we assume that new business pricing is the main driver behind the risk allocation case study. The examination of each assumption and the suggested solutions reflect this focus on pricing. We might have used a different methodology had the purpose of the allocation been different.

#### 4.3.1 Pro Rata Allocation based on Standalone RBC Results

This allocation method uses a very simple pro rata approach with the weights based upon the standalone RBC figures for each class of business. The assumptions that underlie this method as applied here are:

- A1 VaR is the Risk Measure.
- A2 Diversification benefit allocated on a pro rata basis, based on standalone results.
- A3 The time horizon for new business is three years.
- A4 The projection period is the time horizon plus ten years for full run off of reserves.
- A5 Relative profitability of business classes is included.
- A6 Includes market risk.
- A7 The model includes historic reserve risk and new business risk.
- A8 A Risk-Tolerance of 99.8% at Year 1 (minus 0.2% for each subsequent year).
- A9 Capital to be serviced is the RBC amount.
- A10 The model is net of reinsurance.
- A11 Operational risk is excluded.

**Table 4.3.1.1: Capital Allocation**

Class	St'lone RBC €m	Capital €m	Allocation %
Motor	104.5	66.6	51%
Home	63.8	40.7	31%
Liability	35.7	22.8	18%
<b>Total</b>	<b>204.0</b>	<b>130.1</b>	<b>100%</b>

**Table 4.3.1.2: Reconciliation to RBC**

	Capital €m
Standalone RBC	204.0
Diversification Credit	(73.9)
<b>RBC Figure</b>	<b>130.1</b>

This first result illustrates the extent of the diversification credit when running a multi-line company. The diversification credit reduces the capital required by approximately one third. In practice, the pro-rata method is not a good way of allocating capital but it has the advantage of being very easy to apply.

### 4.3.2 Assumptions A1 and A2 [Risk Measure & Allocation Method]

#### *Risk Measure*

VaR is not a coherent risk measure. In addition VaR depends on just one simulation from the model and contains no further information about the tail. The alternative tVaR measure is coherent and uses many simulations therefore using more information around the tail. We can choose the tVaR percentile so that it produces the same overall RBC figure. In our case study this turns out to be 98.16%, in other words out of 10,000 simulations tVaR considers the worst 184 ones.

The allocation in table 4.3.1.1 used the results of each line of business as if they were individual companies in their own right. Therefore, the worst outcomes for each line in isolation were taken, rather than for the company as a whole. In reality the main benefit of diversification comes from the fact that lines of business are not perfectly correlated so that in any one year the worst outcomes for each line of business will not necessarily all happen together. For example, Home may suffer a terrible catastrophic event but that does not mean that the Liability market will simultaneously suffer a huge number of large claims. Therefore, we take the worst 184 outcomes for the company as a whole rather than the worst 184 for each line of business separately. We refer to this as using a “Top-Down Approach”.

*Response: Change Risk Measure to tVaR and allocate from the Top Down*

**Table 4.3.2.1: Capital Allocation**

Class	Capital €m	Allocation %
Motor	79.0	61%
Home	53.6	41%
Liability	(2.5)	(2%)
<b>Total</b>	<b>130.1</b>	<b>100%</b>

Now the allocation of capital is based upon a tVaR risk measure and we are using a top down approach.

At this point the assumptions underlying the allocation are:

- A1            tVaR is the Risk Measure.
- A2            Diversification benefit allocated using a top down approach.
- A3...A11    As stated in §4.3.1 above.

On the face of it this method would appear more sensible. However the results do not bear this out. The amount of capital allocated to Liability is negative which suggests that Liability does not require any capital; in fact it would appear to contribute capital. Common sense would pose the question “how could a long tail and risky insurance product contribute capital?” The next few sections will tackle this and other issues.

A closer look at the simulations that drive the result would reveal that the capital required by Home is due to an accumulation of catastrophic events over the three year period, while motor experienced very poor claims experience on business written. [Refer back to figure 3.4.1 for an illustration of this.]

### 4.3.3 Assumption A3 [Time Horizon]

The base model uses a time horizon of three years for written business and one year earned. Non-Life Insurance is typically written on an annual basis, and return on capital is measured annually.

This raises the question of whether capital allocation should be based upon a three year time horizon to be the same as our technical definition of RBC or should it use a one year time horizon.

On one hand the pricing actuary might argue:

- An annual product will be priced now to take into account the risks that it will face over the next year.
- There is no need to use a longer time horizon since the product can be re-priced during this period.

On the other hand the capital management actuary may argue that:

- There is an underwriting cycle.
- There is a competitive market.
- The rating for some products is more volatile than others. Taking a one year time horizon can not capture this volatility.

For this case study the Pricing Actuary prevails and we will use a one year time horizon. However changing the time horizon to one year will mean that the capital allocated is no longer the same as the company’s RBC figure, although it is still consistent in all other aspects.

*Response: Change time horizon to one year.*

**Table 4.3.3.1: Capital Allocation**

Class	Capital €m	Allocation %
Motor	39.6	47%
Home	31.2	37%
Liability	13.4	16%
<b>Total</b>	<b>84.2</b>	<b>100%</b>

Table 4.3.3.2: Reconciliation to RBC

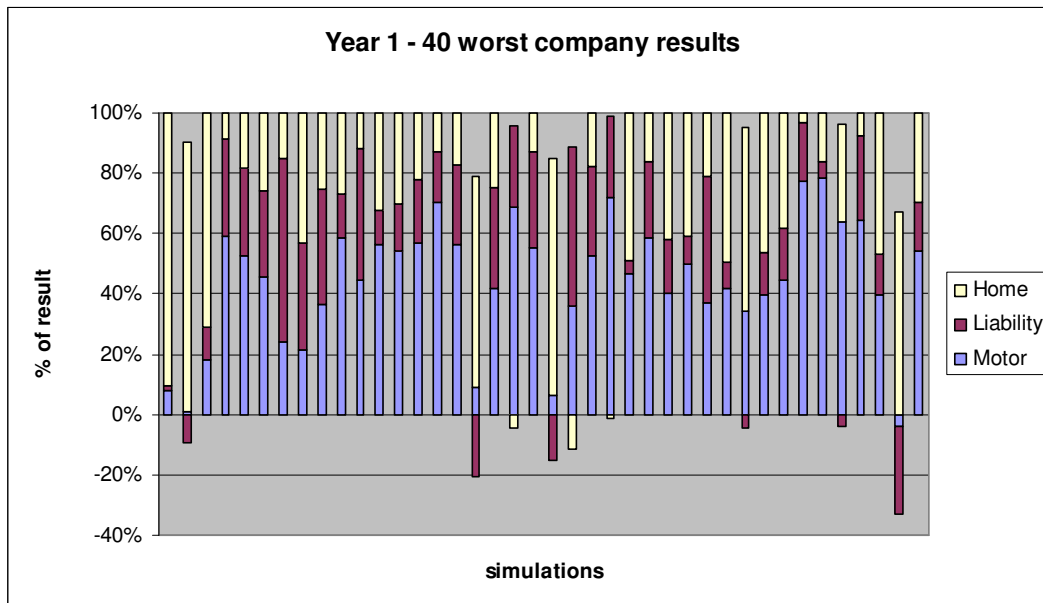
	Capital €m
Allocated	84.2
2 Years Extra Risk	45.9
RBC Figure	130.1

At this point the assumptions underlying the allocation are:

- A1 tVaR is the Risk Measure.
- A2 Diversification benefit allocated using a top down approach.
- A3 The time horizon for new business is one year.
- A4 The projection period is the time horizon plus ten years for full run off of reserves.
- A5...A11 As stated in §4.3.1 above.

Now the allocation of capital is based upon a tVaR risk measure and a one year time horizon. To illustrate the results, a similar graph to figure 3.4.1 is presented below. In this instance, the simulations illustrated are the 40 worst overall company results in year 1.

Figure 4.3.3.1: 40 Worst Simulations in Year 1 by Class of Business



As before each bar represents an individual simulation. For each bar the proportion of the loss caused by each class of business in that simulation is illustrated.

Points illustrated by the graph:

- Motor still requires the most capital followed by Home and Liability respectively.
- More CATs are apparent than the graph in Section 3.4.1. This is simply because the simulations illustrated here are the most extreme.

The proportions of capital allocated to each class over a one year time horizon have moved relative to a three year time horizon.

- The proportion of capital allocated to Liability has increased but it is still much smaller than the other two classes.
- The proportion allocated to Home has remained much the same.
- The increase in the Liability proportion is mostly offset by a reduction in Motor's proportion.

As pointed out the proportion allocated to Liability is much smaller than for the other classes. The main reason for this is that the expected profitability of the Liability business is much larger than the other classes. The next section considers this issue.

### 4.3.4 Assumption A5 [Relative Profitability]

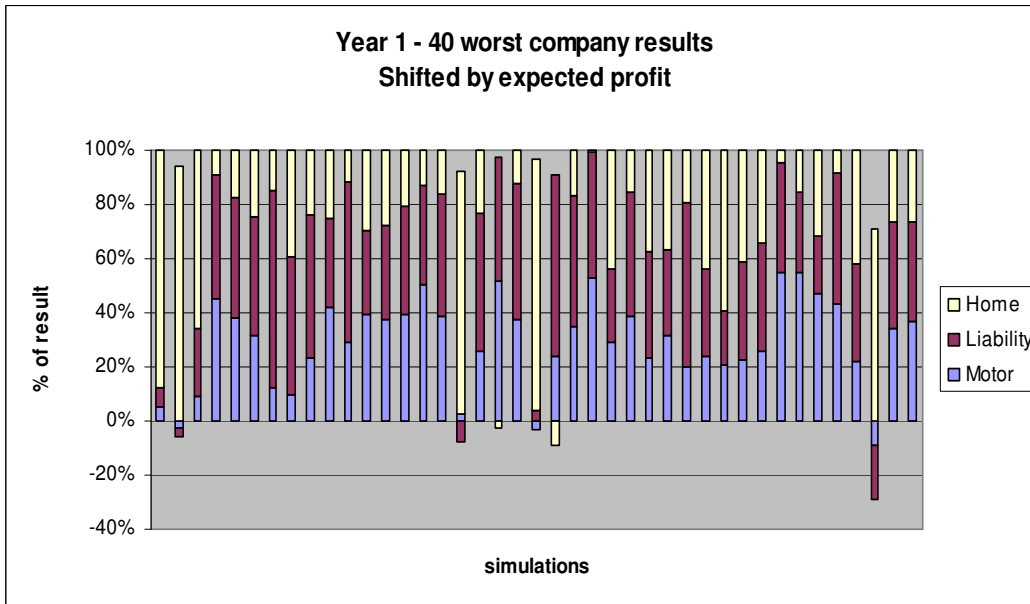
The base model includes the relative profitability of the different lines of business. Without adjustment this can distort the capital required, in fact the result can be a negative capital requirement (or capital contribution) for very profitable lines of business.

A solution is to consider each line of business as being composed of two parts. The first part contains all the claims and expenses and enough premiums to produce an expected profit of zero over one year. The second part contains the remaining excess (or deficit) of premiums. This second part is therefore riskless and is allocated nil capital.

This can be achieved in practice by subtracting expected profit from the simulated profits for the line of business and performing the allocation based on the transformed profits. Note also that this transformation is not a problem because the tVaR measure is coherent.

By applying this transform we are now looking at the actual risks of the product without being clouded by potential short term profits. Figure 4.3.4.1 below illustrates this point.

Figure 4.3.4.1: 40 Worst Simulations in Year 1 - Shifted by Discounted Expected Profit.



This graph is a follow on from Figure 4.3.3.1. For each simulation the expected profit by class has been deducted from the losses. Compared to Figure 4.3.3.1 the proportion of the bars relating to Liability has greatly increased, with a reduction in Motor and a very slight reduction in Home.

Now the results are based upon the variability around a mean profit of zero, rather than the results being distorted by profits or losses. This can be considered as a better measure of risk.

*Response: Remove expected profitability / loss from each line of business,*

Table 4.3.4.1: Capital Allocation

Class	Capital €m	Allocation %
Motor	29.1	30%
Home	32.6	34%
Liability	35.2	36%
<b>Total</b>	<b>97.0</b>	<b>100%</b>

Table 4.3.4.2: Reconciliation to RBC

	Capital €m
Allocated	97.0
2 Years Extra Risk	45.9
Expected Profit	(12.8)
<b>RBC Figure</b>	<b>130.1</b>

At this point the assumptions underlying the allocation are:

- A1 tVaR is the Risk Measure.
- A2 Diversification benefit allocated using a top down approach.
- A3 The time horizon for new business is one year.
- A4 The projection period is the time horizon plus ten years for full run off of reserves.
- A5 Relative profitability of business classes is excluded.
- A6...A11 As stated in §4.3.1 above.

Now the allocation of capital is based upon a company with expected profits of zero, which leaves just the volatility or risk around a mean of zero. The results are looking more reasonable and properly reflect the risks underlying each product. Note that the overall impact on the Allocated figure is to increase it by €12.8m. The expected profits made by Liability are partially offset by the losses incurred by motor.

### 4.3.5 Assumptions A5 and A6 [Market Risk & Reserve Risk]

#### *Market Risk*

The base model includes market risk. We now question whether it is fair to penalise the underwriter for investment management decisions.

*Response: Remove market risk*

**Table 4.3.5.1: Capital Allocation**

Class	Capital €m	Allocation %
Motor	22.1	28%
Home	29.7	37%
Liability	28.3	35%
<b>Total</b>	<b>80.1</b>	<b>100%</b>

**Table 4.3.5.2: Reconciliation to RBC**

	Capital €m
Allocated	80.1
2 Years Extra Risk	45.9
Expected Profit	(12.8)
Market Risk	16.9
<b>RBC Figure</b>	<b>130.1</b>

For our example company the market risk is simply a measure of the volatility of investment returns.

At this point the assumptions underlying the allocation are:

- A1 tVaR is the Risk Measure.
- A2 Diversification benefit allocated using a top down approach.
- A3 The time horizon for new business is one year.
- A4 The projection period is the time horizon plus ten years for full run off of reserves.
- A5 Relative profitability of business classes is excluded.
- A6 Excludes market risk.
- A7...A11 As stated in §4.3.1 above.

### **Reserve Risk**

The base model includes historic reserve risk. Again we might ask whether new business should be required to support the risks generated by business written in the past. The business written in the past should have been self-supporting, so any capital requirement in respect of this business should have already been built up.

*Response: Remove historic reserve risk from the model results.*

**Table 4.3.5.3: Capital Allocation**

Class	Capital €m	Allocation %
Motor	13.5	24%
Home	26.6	48%
Liability	15.9	28%
<b>Total</b>	<b>56.0</b>	<b>100%</b>

**Table 4.3.5.4: Reconciliation to RBC**

	Capital €m
Allocated	56.0
2 Years Extra Risk	45.9
Expected Profit	(12.8)
Market Risk	16.9
Historic Reserve Risk	24.1
<b>RBC Figure</b>	<b>130.1</b>

Note that removing historic reserve risk reduced the proportion allocated to Liability by 7 percentage points. This is as expected since liability has the largest outstanding reserves and there is more uncertainty in the ultimate development of Liability reserves. [See Appendix A - Reserve Risk.]

The table above provides our final recommended method of allocation. We use a coherent risk measure tVaR with a risk tolerance that is consistent with the RBC figure. We have allocated based upon a top down approach and removed the expected profits so that the capital is allocated in proportion to the underlying risk rather than being clouded by short term profits. Finally to please the underwriters and pricing actuaries we have used a one year time horizon and removed market risk and historic reserve risk.

In summary the allocation uses the following assumptions

- A1 tVaR is the Risk Measure.
- A2 Diversification benefit allocated using a top down approach.
- A3 The time horizon for new business is one year.
- A4 The projection period is the time horizon plus ten years for full run off of reserves.
- A5 Relative profitability of business classes is excluded.
- A6 Excludes market risk.
- A7 The model excludes historic reserve risk and includes new business risk.
- A8...A11 As stated in §4.3.1 above.

However, there are some assumptions that have not been challenged yet. These are considered further in §4.4 below.

### 4.4 Points for Discussion

#### 4.4.1 Assumption A8 [Risk Tolerance]

##### Risk Tolerance Level

The base model uses a risk tolerance of 99.8% in year 1 (minus 0.2% for each subsequent year). This risk tolerance level considers extreme downsides only. While this is appropriate for credit rating agencies or financial regulators, we might ask whether it is appropriate for pricing products.

The results could be analysed using different risk tolerances to assess the effects on the capital allocations. The table below gives an indication of what the resulting allocation would look like if we used different risk tolerances.

**Table 4.4.1.1: Capital Allocation**

Class	99%	95%	90%	75%
Motor	25%	30%	32%	32%
Home	44%	33%	29%	28%
Liability	31%	37%	39%	40%
<b>Total</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>	<b>100%</b>

There is a noticeable difference in the amount allocated to Home when we move below the 99% mark. This is because the capital for Home is driven by extremely large catastrophes that blow out the reinsurance cover. When we use lower percentiles the amount allocated falls dramatically as the reinsurance cover is adequate at these levels. This provides a useful illustration of why you may decide not to use very high percentiles when allocating capital for pricing purposes. Note also that the allocation by class settles down to relatively constant levels for lower risk tolerances.

#### 4.4.2 Assumption A9 [Reinsurance]

##### Reinsurance

The base model works with net of reinsurance results. We could ask if it is more appropriate to base the allocation on gross reserves.

**Table 4.4.2.1: Capital Allocation based on gross cashflows**

Class	Capital €m	Allocation %
Motor	10.2	18%
Home	33.2	59%
Liability	12.6	22%
<b>Total</b>	<b>70.8</b>	<b>100%</b>

**Table 4.4.2.2: Reconciliation to RBC using gross cashflows**

	Capital €m
Allocated	70.8
2 Years Extra Risk	45.9
Expected Profit	(12.8)
Market Risk	16.9
Historic Reserve Risk	24.1
Reinsurance	(14.8)
<b>RBC Figure</b>	<b>130.1</b>

The amount of capital allocated to Home has increased significantly. This is due to the impact of catastrophes with no reinsurance protection. Reinsurance is usually an integral part of a product's design structure and it is one of the most common methods used to mitigate risk. For these reasons we did not remove it from our favoured method.

#### 4.4.3 CAT Risk

CAT risk may be considered as quite different in nature from other risks and the methodologies described above may not capture the true cost of the risk. Even if the overall capital figure does capture the CAT risk the methods used to allocate capital may not. The table below provides the result from our example company when CAT risk is removed. It follows on from table 4.3.5.3.

**Table 4.4.3.1: Capital Allocated with CAT Risk Removed**

Class	Capital €m	Allocation %
Motor	15.3	34%
Home	11.0	24%
Liability	19.2	42%
<b>Total</b>	<b>45.6</b>	<b>100%</b>

**Table 4.4.3.2: Reconciliation to RBC with CAT Risk Removed**

	Capital €m
<b>Allocated</b>	45.6
<b>2 Years Extra Risk</b>	45.9
<b>Expected Profit</b>	(12.8)
<b>Market Risk</b>	16.9
<b>Historic Reserve Risk</b>	24.1
<b>CAT Risk</b>	10.4
<b>RBC Figure</b>	<b>130.1</b>

Therefore on a diversified basis the cost of the CAT (net of reinsurance) at the end of year 1 is €10.4m.

On a consistent but standalone basis the CAT risk was calculated as €34.3m.

## 5.0 Reinsurance

We now leave aside capital allocation and in this section describe a possible approach to the analysis of reinsurance arrangements using RBC techniques. The description considers excess of loss treaties but similar approaches can be applied to other reinsurance arrangements.

### 5.1 Model

In the description of a possible ALM reference was made to retaining the results net and gross of reinsurance and maintaining a separate module for reinsurance arrangements. The ability to pass data and assumptions through a reinsurance module with results recorded net and gross will allow the user to analyse different reinsurance strategies.

We will use our example company to illustrate how XOL reinsurance arrangements can be analysed. The key assumptions for this investigation are frequency and severity of large claims. The assumptions used in our example are set out in Appendix A.

We now run as many simulations as possible given the constraints of the model and computer power; again we would recommend at least 10,000.

For each retention limit we record key financials such as; average recovery, spread of results, etc. We can compare these results with the proposed reinsurance premium rates. Table 5.2.1 shows the expected recovery for each line of business at different retention limits.

**Table 5.2.1: Expected Recoveries for Various Reinsurance Limits**

	Liability €m	Motor €m	Home €m
Expected Recovery €1m limit	5.5	13.4	1.4
Expected Recovery €3m limit	2.1	5.0	0.1
Expected Recovery €5m limit	1.1	2.7	0.0
Expected Recovery €10m limit	0.4	1.1	0.0

Clearly the amount of recovery reduces as the limit increases. The expected recoveries from each treaty could be compared to the reinsurance premium being charged. Note that the results above are based upon mean position; we could use different percentiles (e.g. what would the premium be if it were based on 75%).

### 5.2 Reinsurance Strategy

We can also consider the impact upon the company's capital requirements when reinsurance strategies are changed. For example all else being equal by how much does our capital requirement change if we increase the retention limit by €4m from €1m to €5m. This analysis will allow us to assess the balance between giving away profit and reducing the volatility of the result.

For this part of the analysis we need to know the cost of the reinsurance. Then we can comment upon the balance between cost and reduction of capital.

### 6.0 Other Practical Applications

In this section a brief mention is given to some other areas where RBC techniques could be applied to solve real business problems.

#### 6.1 Reserving

Stochastic reserving is becoming a more important tool in the reserving process. The role of stochastic reserving will increase as regulators and reserve setting for financial statements move towards a more risk based regime. The topic of stochastic reserving is a very wide one and we do not propose to go into any detail in this paper.

An integral part of the ALM model is the stochastic reserving module. In the design structure set out in Appendix B, this is highlighted separately. It makes sense building an ALM to keep reserving in mind and vice-versa, so that the two models can dovetail together in a coherent way.

Stochastic reserving normally considers the distribution of future claims payments from a fixed point in time. An ALM allows the evolution of the stochastic reserves themselves to be examined over time.

#### 6.2 Optimal Decision Making

Many of the practical applications described above are essentially optimal decision making strategies. For example, reinsurance problems can be assessed by asking the question, what reinsurance strategy will produce the optimal return on capital for the company? Many problems can be expressed in this way and the RBC model and techniques can be adapted to try to solve them.

Among the types of questions that might be tackled are

- What is the optimum Investment Strategy for a given company?
- What mix of new business by class should be aimed for to minimise RBC requirements, or maximise return on capital?
- Is there an optimal business mix to take advantage of diversification benefit?

- How large does a line of business need to be to produce stable results?
- How best can business be organised to produce a stable profit stream?

### 7.0 Other Practical Issues

This section considers a number of issues that you should consider and will probably face when calculating the RBC or applying the techniques. They range from communication of results to the uncertainty caused by sample error.

#### 7.1 Communication of Results

This is a very important part of any strategy to use RBC techniques within a company. The ideas and methodology behind RBC are complex. It will be a real challenge for actuaries to present the results and findings from a RBC exercise to senior management. Failure to communicate effectively will result in your work not being appreciated or used, or even worse mis-used.

As Solvency II approaches, risk-based techniques will increase in importance. Senior management will become more aware of the requirements to determine capital in more scientific ways. As this paper has tried to point out, a real opportunity will be lost if you simply use the techniques to determine solvency capital. Through effective communication you should be able to convince the senior management of the merits of applying the same ideas to real business problems and strategy.

We believe that the sooner you start the communication process the better. It will be a long journey and will require a lot of patience. Concepts and ideas should be explained before results are presented. Take small steps rather than trying to explain it all in one go, for example the following steps may be taken in your communication journey:

- Describe risks that are covered by the RBC figure.
- Explain concept of volatility.
- Describe how the assumptions are set, stressing the importance of realism.
- Describe how the model was constructed to cover the risks.
- Explain VaR and tVaR.
- Explain the concept of diversification.
- Describe how you bring all of the above together to determine the RBC figure.

At this point your audience will hopefully understand the basic concepts and what the RBC figure represents. Now you can continue the journey to explain the following:

- Allocation of capital to class of business.
- Using the techniques to inform reinsurance strategy.
- Using the techniques to inform other strategic decisions such as long term investment policy.

### 7.2 Parameter / Model Risk

The results from a RBC exercise are dependent upon the model used and the assumptions fed into it. You must ask yourself the questions:

- Does our model truly reflect the dynamics of the company?
- Do the assumptions reflect the true underlying distributions of the risks being modelled?

These questions will always be posed when you are performing any modelling exercise. But it is of particular concern when the model is being used to calculate a RBC figure. RBC modelling necessarily deals with extreme cases or the tail of possible outcomes. It is very difficult to parameterise the tail accurately. Traditional methods will tend to underestimate the potential worst outcomes.

In our model we have used a couple of techniques to attempt to reduce parameter error.

- Claims are split into large and attritional. Extreme Value Theory was used to parameterise the severity of large claims.
- For frequency of large claims a Bayesian technique was used. A prior distribution was used (Gamma) to determine the mean number of claims. This mean was then used as the parameter in the posterior distribution (Poisson).

### 7.3 Sample Error

Sample error can be explained by a simple example. We ran our model six times (including the run used in the paper) with the same assumptions and inputs but with different seed values for the random number generator. The RBC figure for the total company and on a standalone basis for each line of business is given in the table below.

**Table 7.3.1: Sample Error**

Run No.	Total RBC €m	St' lone Motor €m	St' lone Home €m	St' lone Liability €m
1	130.1	104.5	63.8	35.7
2	130.8	102.2	63.0	36.6
3	133.1	103.7	63.3	33.4
4	131.8	105.3	63.8	36.2
5	130.8	101.3	64.0	35.2
6	132.6	110.2	60.1	40.4

**Table 7.3.2: Analysis of Sample Error**

	Total RBC €m	St' lone Motor €m	St' lone Home €m	St' lone Liability €m
Average	131.5	104.5	63.0	36.263.0
Max	133.1	110.2	64.0	40.4
Min	130.1	101.3	60.1	33.4
Max - Min	3.0	8.9	3.9	6.9
Diff as % of Avg	2.3%	8.5%	6.2%	19.1%

In our short analysis sample error is small for the overall RBC but it can be quite high when looking at individual line of business.

Increasing the number of simulations can reduce sample error. But there will be a practical limit to the number of simulations that can be run. It is important to know that sample error will exist and you should try to understand and estimate the extent of it.

## 7.4 Operational Risk

This paper has hardly mentioned Operational Risk except to say that it is one of the risks that must be allowed for under Solvency II. The model that we used did not allow for Operational Risk, but it does exist and your final capital figure should include it.

## 7.5 Behavioural and Dynamic Feedback Effects

How is the capital required affected by human actions or behaviour? This question is important but very difficult to answer. Examples are:

- Management actions.
- Reserving cycle.
- Underwriting cycle.
- Elasticity of demand.
- Cost of reinsurance.

All of the above are directly affected by the behaviour and actions of our key stakeholders. For example, senior management can take decisions that will reduce or increase risk, our customers will determine how much extra business we may write given a change to premium rates.

Our model does not include the impact of actions or behaviour. With more work you could include:

- Dynamic decision making rules.
- Assumptions for an underwriting cycle.
- Assumptions for elasticity of demand.
- A model for changes in the cost of reinsurance in line with the underwriting cycle.

If management actions were included you would have to demonstrate that these actions were embedded in the company.

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## Appendix A: Example Company

### A.1 Introduction to Example Company

The results in the paper are based upon an example insurance company that writes three lines of business, namely:

- Motor
- Home
- Liability

The company is structured so that the premium income for each class of business is approximately the same. However, the underlying experience and drivers of profit are quite different across the classes of business.

#### *Correlation*

The model assumes a strong correlation (0.8) between the Motor and Liability classes, whereas Home is not correlated to the other classes. This assumption is summarised in the table below.

**Table A.1.1: Correlation Matrix**

	Motor	Home	Liability
Motor	1.0	0.0	0.8
Home	0.0	1.0	0.0
Liability	0.8	0.0	1.0

The correlations are applied to the total cost of claims excluding large claims (see later for the definition of large claims).

### A.2 Premium and Exposure

**Table A.2.1: Average Premium per Policy (€)**

Class	Prior Year	Year 1	Year 2	Year 3
Motor	477	477	496	516
Home	476	476	464	482
Liability	4,793	4,793	4,404	4,624

**Table A.2.2 - Business Volumes Written (& Earned from Prior Year)**

Class	Prior Year	Year 1	Year 2	Year 3
Motor	90,000	200,000	220,000	240,000
Home	90,000	200,000	220,000	240,000
Liability	10,000	20,000	22,000	24,000

**Table A.2.3: Written Premium & Earned from Prior Year (€m)**

Class	Prior Year	Year 1	Year 2	Year 3
Motor	42.9	95.4	109.1	123.8
Home	42.8	95.1	102.0	115.8
Liability	47.9	95.9	96.9	111.0

### **Underwriting Noise**

Underwriting noise is modelled by assuming a degree of noise around the mean loss ratio for each line of business. The noise is modelled by a Normal distribution. The standard deviations for each class of business are provided in the table below.

**Table A.2.4: Underwriting Noise**

Class	Std Deviation
Motor	10.0%
Home	7.5%
Liability	15.0%

### Other Related Assumptions

- It is assumed that premiums are earned evenly over the year.
- Rating changes take place at the end of the calendar year.

## **A.3 Expenses and Commission**

Expenses and commission are expressed as a percentage of premium.

**Table A.3.1: Expenses**

Class	Unearned	Year 1	Year 2	Year 3
Motor	10%	10%	10%	10%
Home	10%	10%	10%	10%
Liability	10%	10%	10%	10%

**Table A.3.2: Commission**

Class	Unearned	Year 1	Year 2	Year 3
Motor	10%	10%	10%	10%
Home	10%	10%	10%	10%
Liability	15%	15%	15%	15%

### Related Assumptions

- 40% of total expenses and commission can be held as a Deferred Acquisition Cost (DAC) Asset.
- When the business goes into run off the expenses are set at €9m split evenly across the three lines of business.

#### A.4 Claims Assumptions

##### *Large Claims - Severity*

The severity of large claims is modelled with the Generalised Pareto Distribution.

**Table A.4.1: Large Claims Severity**

Class	Threshold	Shape	Scale	Mean	St Dev
Motor	1,000,000	0.40	800,000	2,335,965	2,717,543
Home	500,000	0.05	500,000	1,027,643	557,442
Liability	500,000	0.40	600,000	1,492,066	2,056,055

##### *Large Claims - Frequency*

The number of large claims is modelled using a Poisson distribution with mean per million policies. The mean is produced by a Gamma Distribution. For each simulation the gamma distribution will produce a frequency per million which is then used as the parameter in the Poisson distribution.

**Table A.4.2: Large Claims Frequency : Parameters of Gamma Distribution**

Class	Alpha	Beta	Mean	St Dev
Motor	4.0	10.00	40	20
Home	4.0	7.50	30	15
Liability	9.0	33.33	300	100

##### *Attritional Claims - Severity*

The average cost of attritional claims is modelled using a Gamma distribution.

**Table A.4.3: Attritional Claims Severity**

Class	Alpha	Beta	Mean	St Dev
Motor	50	50	2,500	353.5
Home	300	10	3,000	173.2
Liability	125	350	43,750	3,913.1

##### *Ex- Large Claims - Frequency*

The frequency of ex-large claims is modelled using a Gamma distribution. The frequency is expressed as a percentage of the exposure policy count.

**Table A.4.4: Ex-large Claims Frequency**

Class	Alpha	Beta	Mean	St Dev
Motor	150	0.00080	12.0%	0.98%
Home	50	0.00175	8.75%	1.24%
Liability	40	0.00075	3.0%	0.47%

**Catastrophe**

It is assumed that the only class of business impacted by catastrophes is Home.

**Inflation**

Expense inflation is stochastically simulated within the asset model. Claims inflation is set equal to expense inflation plus a constant that is dependant upon class to produce a higher average inflation figure. The table below provides the average claims inflation for each class of business.

**Table A.4.3: Mean Claims Inflation**

Class	Mean Claims Inflation
Motor	4.0%
Home	4.0%
Liability	5.0%

**Loss Ratio**

The average loss ratio in year 1 for each class of business taken from the actual model run is given in the table below.

**Table A.4.3 - Average Net Loss Ratio**

Class	Year 1
Motor	98%
Home	71%
Liability	43%

**A.5 Balance Sheet Data**

**Statutory Balance Sheet**

The balance sheet below is the statutory position for the example company at the start of the projection. The balance sheet is split by line of business. The split by line of business is determined as follows:

**Assets**

- Investible assets      Split in proportion to statutory net reserves
- Reinsurance asset      Calculated at line of business level
- Other assets              Split in proportion to statutory net reserves
- DAC                          Calculated at line of business level
- Goodwill                  Split in proportion to statutory net reserves

**Liabilities**

- UPR                          Calculated at line of business level
- Outstanding claims      Calculated at line of business level
- Shareholder Funds      Balance of Assets less Reserves

**Table A.5.1: Statutory Balance Sheet Split by Class of Business (€m)**

Balance Sheet Item	Motor	Home	Liability	Total
Investible Assets	236.3	120.7	371.8	728.8
Reinsurance Asset	8.8	0.0	14.2	23.0
Other Assets	4.3	2.2	6.8	13.3
DAC	7.7	7.6	9.6	24.9
Goodwill	3.2	1.7	5.1	10.0
<b>Total Assets</b>	<b>260.3</b>	<b>132.2</b>	<b>407.5</b>	<b>800.0</b>
UPR	42.9	42.8	47.9	133.6
Outstanding Claims	185.5	69.4	311.8	566.7
Shareholder Funds	31.9	20.0	47.8	99.7
<b>Total Liabilities</b>	<b>260.3</b>	<b>132.2</b>	<b>407.5</b>	<b>800.0</b>

***Converting Statutory into Realistic***

The RBC figure is determined by projecting realistic balance sheets. The methodology used to determine the realistic position is described below. Table A.5.2 sets out the realistic balance sheet at the start of the projection and how it compares to the statutory position.

**Assets**

- Investible assets No change since taken at market value
- Reinsurance asset Discounted using risk free rate, no surplus
- Other assets No change
- DAC Removed since it is an intangible asset
- Goodwill Removed since it is an intangible asset

**Liabilities**

- UPR Surplus removed and discounted
- Outstanding claims Surplus removed and discounted
- Shareholder Funds Balance of Assets less Reserves

**Table A.5.2: Conversion of Statutory to Realistic Balance Sheet (€m)**

Balance Sheet Item	Total Statutory	Total Realistic
Investible Assets	728.8	728.8
Reinsurance Asset	23.0	21.4
Other Assets	13.3	13.3
DAC	24.9	0.0
Goodwill	10.0	0.0
<b>Total Assets</b>	<b>800.0</b>	<b>763.5</b>
UPR	133.6	89.1
Outstanding Claims	566.7	486.1
Shareholder Funds	99.7	188.3
<b>Total Liabilities</b>	<b>800.0</b>	<b>763.5</b>

The assumptions underlying the conversion of the balance sheet from a statutory position to a realistic position are as follows:

- The reinsurance asset, UPR and outstanding claim reserves are discounted using a risk free rate. Gross and net cashflows are

projected and the appropriate term structure of interest rates is applied to these cashflows to obtain the discounted values. The equivalent aggregate discount factor for the gross outstanding claims is applied to the UPR.

- Surplus in the outstanding claims is removed. The assumed surplus as a percentage of reserves is given in the table below:

**Table A.5.3: Initial Surplus as % of Outstanding Claim Reserve**

Class	Surplus in Outstanding Claims
Motor	5.0%
Home	5.0%
Liability	10.0%

- Expected profit/loss in the UPR is removed/added. The expected profit or loss is dependant upon the loss ratio for each class of business.
- Intangible assets such as DAC and goodwill are removed. Any allowance for DAC within the UPR is also removed.

### A.6 Investment Strategy

The company has one fund for investment purposes. The asset mix for this fund is given in the table below.

**Table A.6.1 : Investment Strategy**

Asset Type	Mix
Cash	10%
Bonds	75%
Equity	10%
Property	5%

The asset mix is rebalanced at each year end back to the stated investment strategy.

### A.7 Reserves

Cumulative payment triangles were set up for each class of business. The volatility and payment pattern are different for each class of business. There is no assumed correlation between the different classes in respect of reserving.

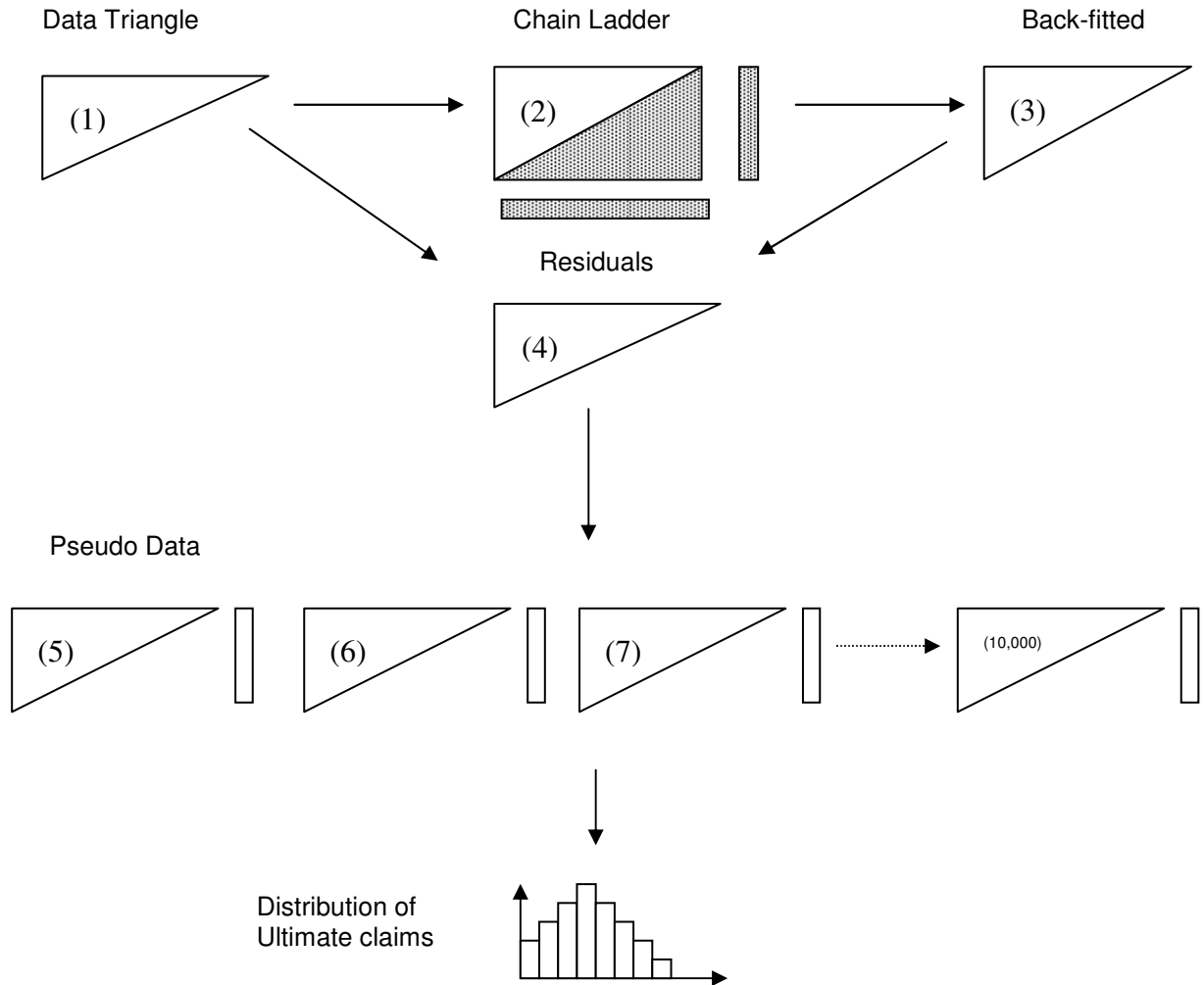
Although there is a substantial sub-model to generate the historic reserve risk it can be thought of in the same way as us supplying 10,000 scenarios of the possible run-off of the current reserves. The average future run-off will equal the best estimate reserve. It also produces the projected payment patterns necessary to model the run off of the future new business as we progress through the projection.

The technique used to generate the 10,000 scenarios of reserve run off for the current book is 'Bootstrapping'. In this instance we have assumed that the underlying distribution is the Over-Dispersed Poisson (ODP).

## Bootstrapping

What follows is a very simplified explanation of bootstrapping but it should give some idea of the process. There are numerous statistical papers backing the methodology and it is widely used by practitioners.

Essentially it is a mechanism to generate a distribution of future run-off outcomes for the current booked reserves based on the inherent noise in the triangle relative to the Basic Chain Ladder development factors.



- (1) Data triangle of Paid Claims
- (2) Calculate the Basic Chain Ladder development factors implied by the triangle
- (3) Create a 'perfect' triangle by back-fitting the ultimate claims across the triangle by using the development factors. This is the 'perfect' triangle as implied by the Chain Ladder factors.
- (4) The residuals are calculated by comparing the fitted triangle with the actual one. Our model uses the Pearson Residual. This is calculated as:

$$\frac{(Actual - Expected)}{\sqrt{(Expected)}}$$

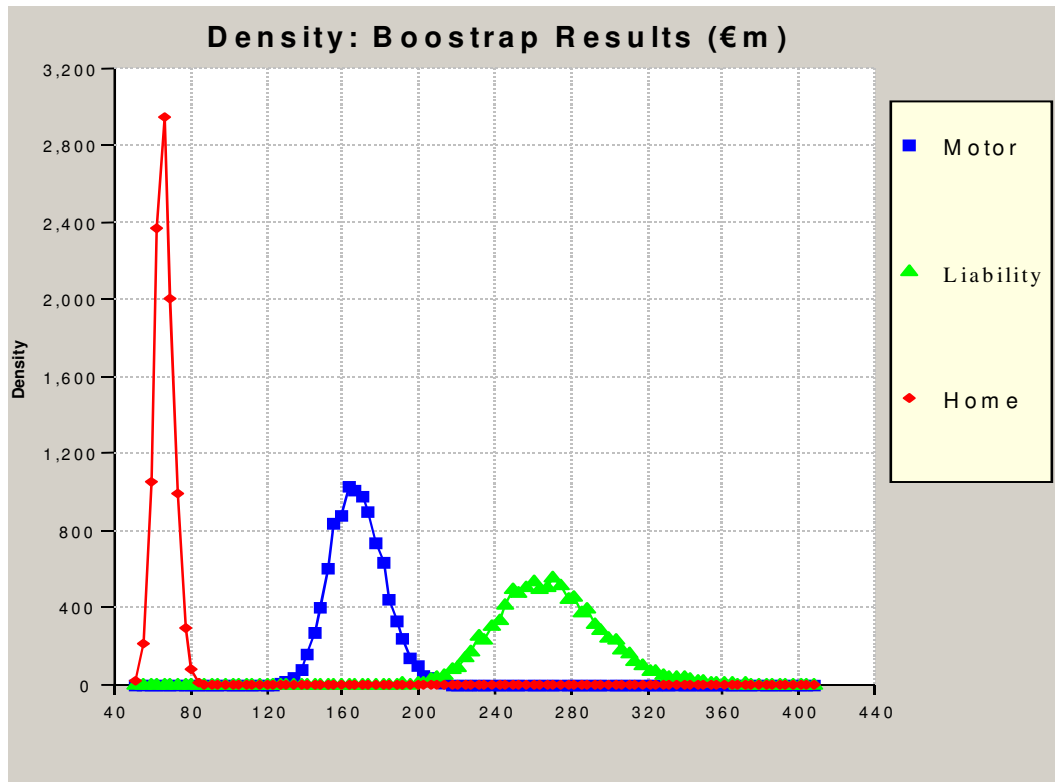
(5) The actual triangle does not match this ‘perfect’ triangle and represents one possibility of deviation from the perfect triangle. We can create other views of the ultimate outcome by randomly re-sampling with replacement the residuals. The resulting residuals can then be combined to the observed (original) triangle to create another triangle giving another view of the ultimate claims.

We do this 10,000 times to get 10,000 views of what the ultimate claims should be and the future claims payments associated with these ultimates. These 10,000 sets of claims payments and ultimates are then sent further into the model in the same way as the CATs and Asset Scenarios. (Note we scale the future claims payments so that on average they equal our view of the best estimate, which are not based on a pure paid BCL method).

Table A.7.1: Reserve Uncertainty

Class	Standard Deviation
Motor	8.4%
Home	7.4%
Liability	10.4%

Figure A7.1: Distribution of Ultimate Values From Bootstrap



The greater reserve risk is illustrated by the greater spread of ultimate values around the mean - in this instance Liability having the greatest spread.

## A.8 Reinsurance

The example company has two forms of reinsurance:

- Individual XOL treaty
- CAT treaty

The details of the treaties are summarised in the table below.

**Table A.8.1: XOL Treaty**

	<b>Motor</b>	<b>Home</b>	<b>Liability</b>
<b>Excess</b>	€1,000,000	€1,000,000	€1,000,000
<b>Limit</b>	Unlimited	€40,000,000	€40,000,000
<b>Free reinstatements</b>	20	20	20

Treaty limits are indexed in line with wage inflation, taken from the ESG.

**Table A.8.2: CAT Treaty**

	<b>Motor</b>	<b>Home</b>	<b>Liability</b>
<b>Excess</b>	N/A	€7,000,000	N/A
<b>Limit</b>	N/A	€160,000,000	N/A
<b>Free reinstatements</b>	N/A	0	N/A

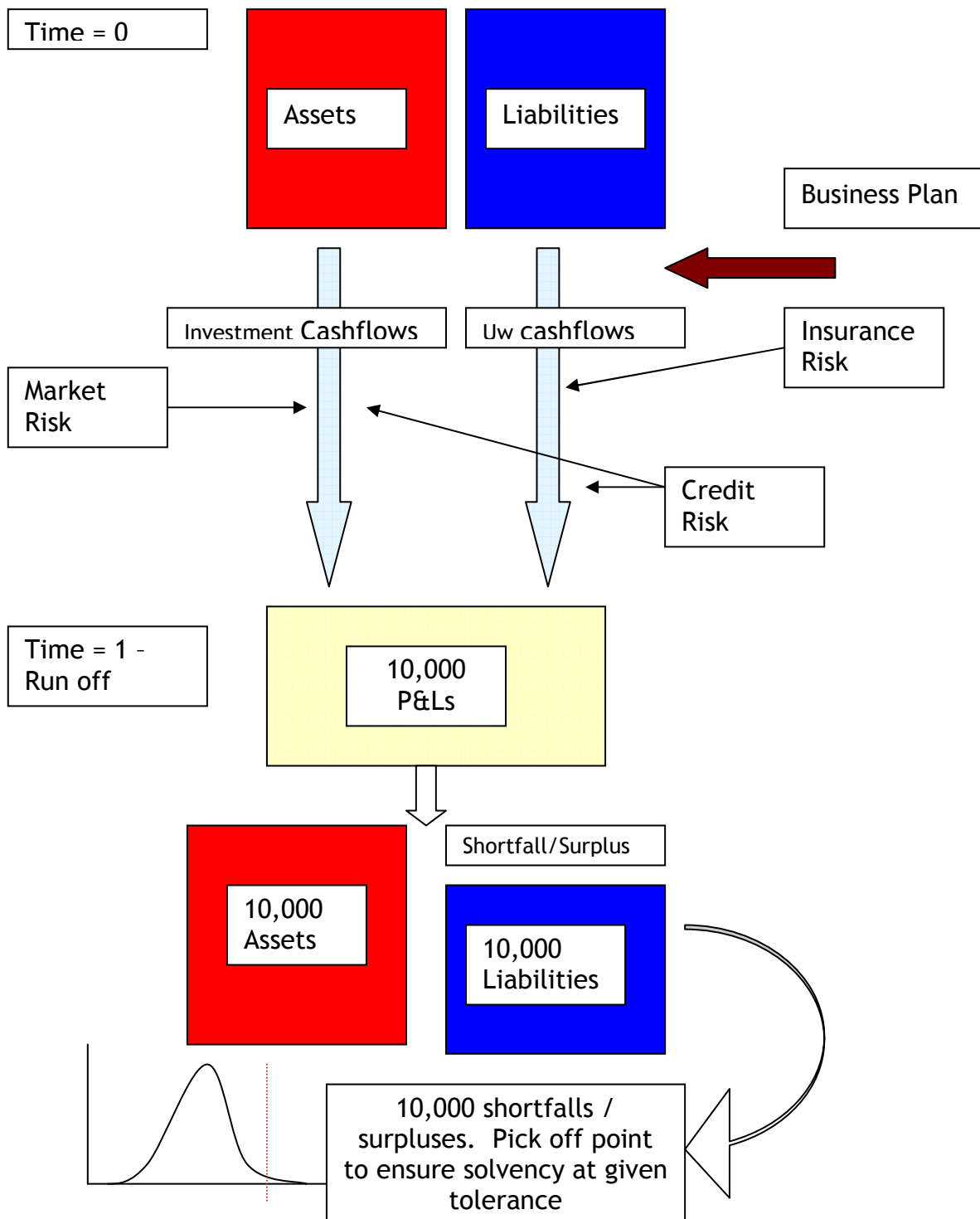
## Appendix B: Model Design

### B.1 Introduction

This appendix discusses how an ALM might be designed. A key consideration in ALM design is the range of purposes to which it is intended the model will be put. This theme will frequently be referred to below.

Figure B.1.1 shows a high level diagram of how a model might fit together.

Figure B.1.1 : High Level Model Design



The model is composed of a series of sub-models or modules each of which we will consider briefly below.

### **B.2 The Model in More Detail**

The model described below will require a large range of assumptions. As these have been referred to in the description of the example company in Appendix A we will not work through them here again except to refer to the assumptions where required in the description of any particular module.

It is also assumed that sitting behind the whole ALM is a suitable random number generator, which can feed random numbers as required to the various modules. An important aspect of model design is that individual scenarios are reproducible if required. This can be achieved by the use of a suitable seeding mechanism for the random number generator.

#### **B.2.1 Economic Scenario Generator**

This is a “black-box” which provides a set of internally consistent scenarios for future income and gains for bonds, equities, property, cash, exchange rates, yield curves, inflation etc. This is the driver behind the whole ALM as it will potentially interact with each other module, e.g. inflation will affect the ultimate outcome for past claims and future claims, and possibly the indexation of reinsurance limits.

#### **B.2.2 Past Claims (Reserves Module)**

This module should simulate the ultimate claims and payment patterns for all prior accident years and generate stochastic claims payment patterns. Bootstrapping is one possible technique that can be used to generate these patterns.

The stochastic payment patterns from the past claims module can be also be employed in the future claims module.

#### **B.2.3 Future Claims**

The future claims module would typically be split into three parts:

##### **B.2.3.1 Attritional Losses**

Attritional losses refer to all claims other than those defined as large claims or claims forming part of a catastrophe. The attritional loss model allows for the modelling of claim frequency, severity and correlations.

Note that this could involve two different types of correlation, correlations across classes, and correlations within classes (frequency versus severity)

The outputs of the attritional losses module are then applied to projected exposures to provide the actual to get losses.

### **B.2.3.2 Large Claims**

The large loss module is similar in concept to the attritional loss module. However correlations might not be modelled in this case.

### **B.2.3.3 Cat Claims**

This module produces projections of future CAT losses given company exposure. The risk of reinsurer default may be linked to CAT events.

### **B.3 Reinsurance Module**

In the reinsurance module the rules behind the reinsurance treaty can be applied to provide net of reinsurance cashflows.

### **B.4 Combined Projection Module**

The combined projection module brings together all the outputs of the modules described thus far.

- Written premium and exposure are applied to earnings patterns to calculate earned values.
- Claims Payment Patterns are applied to the outputs of the past claims and future claims modules to project future reserves.
- Claims Inflation can be applied to the basic claims cashflows.
- Discounted reserves (net and gross) are calculated, as they are required for realistic Balance Sheet.

The output of the combined projection module is the cashflows before investment return for each projection period.

### **B.5 Asset Model**

The asset model takes the outputs of the combined projection module and builds up projection of the evolution of assets over time allowing for the initial investment mix and the model's investment strategy (e.g. rebalancing of assets to maintain the specified investment mix).

This will allow the production of full Profit and Loss Statements and Balance Sheets

The model needs to be able to produce both a realistic and a statutory set of results. For capital allocation to class the P&L's and Balance Sheets need to be a class of business level.

## Appendix C: Risk Measures and Capital Allocation Methods

### C.1 What is a Risk Measure?

Insurance is an inherently risky business. This riskiness encompasses the volatility of claims, and investment returns, the uncertainty about claims reserves and premium adequacy, the prospects for future inflation and business volumes etc. A risk measure is an attempt to quantify this riskiness. Mathematically it is a function that distils all the risks in the business down to one number.

### C.2 Some Examples of Risk Measures

The examples below are just some of the many possible choices for risk measure.

#### *Standard Deviation of Insurance Result*

This is a simple measure of the variability of the insurance result. In the context of insurance the distribution of outcomes tends to be strongly skewed, so the usefulness of this measure is limited.

#### *VaR (Value at Risk)*

VaR is a risk measure very commonly used in the banking industry. The definition of VaR depends on two parameters, the time  $t$  and the percentile  $p$ . The VaR is the  $p^{\text{th}}$  percentile of the distribution of outcomes at time  $t$ .

For example VaR could be the 99.5<sup>th</sup> percentile of the distribution of insurance results at time  $t=1$  (or the end of year 1).

#### *tVaR (tail VaR)*

tVaR is related to VaR and is defined as the expected loss in excess of the  $p^{\text{th}}$  percentile of the distribution of outcomes at time  $t$ .

$$E(X \mid X \geq p^{\text{th}} \text{ Percentile})$$

tVaR is also known as *Tail Conditional Expectation (TCE)*

#### C.2.1 Properties of a suitable Risk Measure / Allocation Methodology

A Risk Measure is the tool we use to make our Capital Allocation. For example, we might choose the standard deviation of the insurance result for a given line of business as a Risk Measure. This may or may not be a sensible choice (see below). However it does encapsulate the riskiness of the line of business as a single number.

There are some basic common sense rules that we might expect any Risk Measure, or method of allocating risk capital to conform to. Many suggested Risk Measures may fall short in some respects and therefore we may raise questions as to their suitability.

### C.2.2 Coherence of Risk Measure

In the literature a Risk Measure is said to be *coherent* if it satisfies four basic properties. (In the definitions below  $X$  and  $Y$  denote the value of particular lines of business, and  $\text{Risk}(\cdot)$  is the Risk Measure.)

#### Sub-Additivity

$$\text{Risk}(X+Y) \leq \text{Risk}(X) + \text{Risk}(Y)$$

The riskiness of two lines of business combined is no greater than the sum of their individual riskiness. Note that for two perfectly correlated lines of business the inequality becomes an equality. At the other extreme two completely uncorrelated lines of business could cancel out each other's risks.

#### Monotonicity

$$\text{If } X \geq Y \text{ then } \text{Risk}(X) \leq \text{Risk}(Y)$$

If a line of business is always worth more than another, in all scenarios, it cannot be riskier.

#### Positive Homogeneity

$$\text{Risk}(kX) = k\text{Risk}(X), \text{ for any constant } k$$

Scaling a line of business by a constant will change the risk by the same proportion.

#### Translation Invariance

$$\text{Risk}(X+k) = \text{Risk}(X), \text{ for any constant } k$$

Adding a risk free line of business to an existing line of business brings no additional risk.

### C.2.3 Coherence of Capital Allocation

A Capital Allocation methodology can be defined as *coherent* if it satisfies the following properties. (In the definitions below  $X$  and  $Y$  denote particular lines of business, and  $\text{Cap}(\cdot)$  is the Capital Allocated on a standalone basis,  $\text{Cap}^*(\cdot)$  is the Capital Allocated when lines of business are combined.)

### No Undercut

$$\text{Cap}^*(X) + \text{Cap}^*(Y) = \text{Cap}(X+Y) \leq \text{Cap}(X) + \text{Cap}(Y)$$

(i)  $\text{Cap}^*(X) \leq \text{Cap}(X)$  [by symmetry]

(ii)  $\text{Cap}^*(X) \geq \text{Cap}(X+Y) - \text{Cap}(Y)$

So  $\text{Cap}(X+Y) - \text{Cap}(Y) \leq \text{Cap}^*(X) \leq \text{Cap}(X)$

The allocation to any line of business should be at least as great as its marginal (“last in”) allocation, and no greater than if it was considered on a stand-alone basis.

### Symmetry

If the risk of two lines of business is the same (as measured by the Risk Measure), the allocation should be the same for each.

### Riskless Allocation

The Capital Allocated to a riskless line of business is nil.

*It is important to note that a Capital Allocation method will not be coherent unless the corresponding Risk Measure chosen is coherent.*

Note also that of the three risk measures described above only tVaR satisfies the coherence property.

### C4.4 How to Allocate Diversification Benefit

The literature on capital allocation methodologies has grown rapidly in recent years. Kaye (2005) gives a good survey of methods some of which are described briefly below.

#### Standalone Allocation (Independent “First In”)

This method assumes that the diversification benefit is shared in proportion to the independent stand-alone capital requirements. This is unlikely to be appropriate because there will likely be some correlations between the various lines of business which need to be taken into account.

#### Marginal “Last-In”

The marginal method considers the capital required by the company with and without each line of business. The difference between these two values is the “marginal” capital required for that line of business. The total capital is then allocated to each line of business in proportion to its marginal capital requirement. This method is at the opposite end of the scale to the standalone method.

### **Shapley Method**

The Shapley Method is based on the average of the “1<sup>st</sup> in”, “2<sup>nd</sup> in” ... “nth in” ... and “last in” values. No scaling is necessary. The method naturally produces an allocation that adds up to the full company RBC. However the method is computationally inefficient because the capital requirement of all the possible combinations of lines of business must be calculated. No small feat for even a few lines of business!

A further problem with the Shapley method is that if one line of business is further subdivided then the allocation to other lines of business may also change. This is not an ideal property for an allocation method.

The Aumann-Shapley method overcomes these shortcomings

### **Aumann-Shapley**

Theoretically the Aumann-Shapley method is based on the rate of increase in the capital allocation to a line of business, for a tiny increase in size in that line of business. This approach overcomes the problems with splitting lines of business. The Aumann-Shapley allocation is also very practical to implement using simulation techniques. (See Ruhm and Mango, who implement the Myers- Read method, which is a variant of Aumann-Shapley).

Kaye notes that a number of academics and practitioners have come separately to the same conclusion - that this is the most appropriate and practical method for risk allocation.

The method used in our case study is effectively a variant on the Ruhm and Mango Approach. If the risk measure chosen is based on the  $p^{\text{th}}$  percentile, then the simulation corresponding to that percentile is examined. The capital required by each line of business in that simulation is the capital allocated to that line of business. We refer to this as a simple “Top Down” approach. Note that with the tVaR risk measure the need for averaging around the tolerance percentile to remove simulation error is reduced.